There are 2 problems in the quiz.

(10 points) 1. For each of the initial value problems, determine if the problem has a unique solution, no solution, or infinitely many solutions in a neighborhood of every point of the initial curve in the domain $x > 0, \ y > 0$. Write down the solution $u(x, y)$ of each problem.

a) $2xu_x - yu_y + u = 1, \quad u(x, 1/x) = 0$;  
b) $2xu_x - yu_y + u = 1, \quad u(x^2, 1/x) = 0$;

c) $2xu_x - yu_y + u = 1, \quad u(x, 1/x) = 1$;  
d) $2xu_x - yu_y + u = 1, \quad u(x^2, 1/x) = 1$.

(12 points) 2. a) Make a transformation of coordinates to obtain the canonical form of the differential equation $(y^2 + 1)u_x + y^2u_y - xu = y^2 - xy$ in the domain $y > 0$.

b) Write down the general solution of the differential equation by solving its canonical form.

c) Verify that the initial value problem $u(x, x) = 1$ for the given differential equation has a unique solution in a neighborhood of every point of the initial curve in the domain $y > 0$.

d) Write down the solution $u(x, y)$ of the initial value problem.