There are 5 problems and a bonus problem in the quiz.

(4 points) 1. a) Find the solution of the initial value problem \( \frac{dx}{dt} = \cos^2x, \quad x(0) = \pi. \)

b) Find the solution of the initial value problem \( \frac{dx}{dt} = \cos^2x, \quad x(0) = \pi/2. \)

(4 points) 2. Check that the curve given by \( x(t) = t^2, \ y(t) = t^4, \ z(t) = t^3, \ t > 0, \) is an integral curve of the vector field \( (2z/x, 4xy/z, 3z^2/y) \) in the domain \( \{(x, y, z) : x > 0, \ y > 0, \ z > 0\} \).

(4 points) 3. Check that the plane \( \{x = 0\} \) is an integral surface of the vector field \( (yz \sin x, xy + xz + yz, 1 + x^2) \).

(4 points) 4. Prove that the equations \( xy + z = c_1, \ x + yz = c_2 \) describe the collection of all integral curves of the vector field \( (xy - z, 1 - y^2, yz - x) \) in the domain \( \{(x, y, z) : y > 1\} \).

(4 points) 5. Find all vector fields in the domain \( \{(x, y, z) : x > y > z > 0\} \) with first integrals \( u_1 = xyz \) and \( u_2 = xy + xz + yz \)

(3 points) Bonus. a) Find all solutions of the initial value problem \( \frac{dx}{dt} = \sqrt{1 - x^2}, \quad x(0) = 0. \)

b) Find all solutions of the initial value problem \( \frac{dx}{dt} = \sqrt{1 - x^2}, \quad x(0) = 1. \)

Keep in mind that \( |x| \leq 1. \)