DEPARTMENTAL FINAL EXAMINATION
Fall 2010

MATH-M 119
BRIEF SURVEY OF CALCULUS

Directions

- **DO NOT OPEN** this test booklet until you are asked to do so.
- There are seven pages on this exam with 20 problems – You MUST get a new exam from the proctor if your exam is incomplete.
- PRINT your name and student ID# and check your section below.
- You have two hours to complete this examination.
- No scratch paper – if you need extra paper use the back of a test booklet page.

NO notes, books, nor graphing calculators allowed.
Cell phones should be OFF. Earpieces are not permitted.

NEATNESS COUNTS. CORRECT NOTATION COUNTS.

To receive credit show supporting work.

Do Not Write In This Area		
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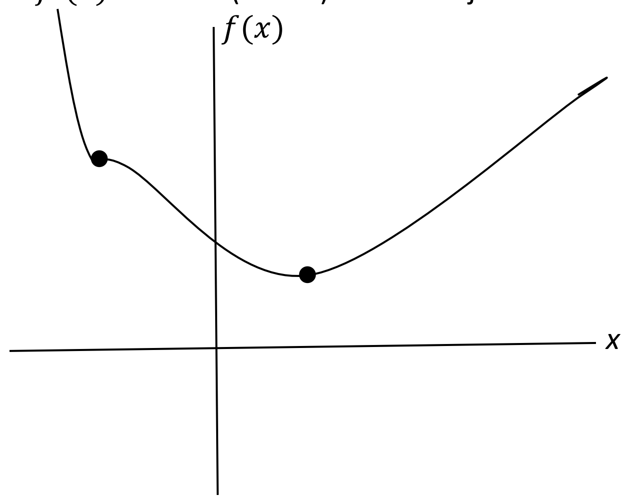
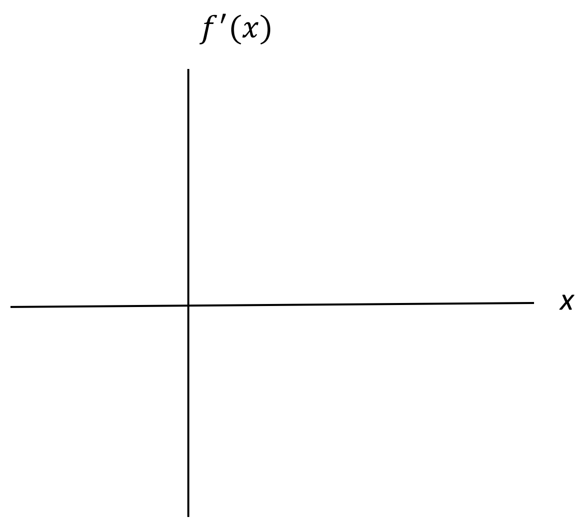
TOTAL	(50)	

NAME (Print Clearly)	
UNIV ID#	

check your section here

1. Sketch a graph of the first derivative $f'(x)$ for the function $f(x)$ depicted below. Make certain that your sketch shows *x*-intercepts exactly where you want them to be. Also make sure your graph of $f'(x)$ is above (below) the *x*-axis just when it's supposed to be.



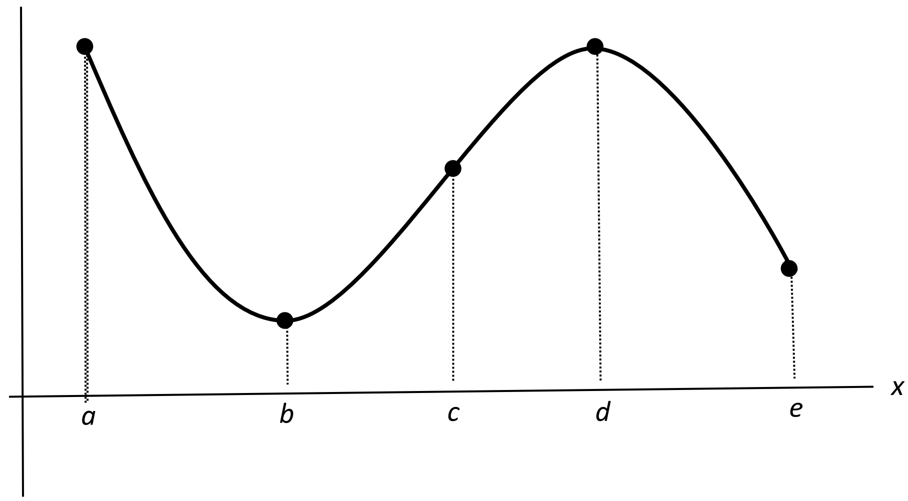


(2)

2. Suppose that $f(x)$ is a function with $f(100) = 52$ and $f'(100) = 0.5$
Use a local linear approximation to estimate $f(96)$

2. $f(96) \cong$ ______________________ (2)

3. Refer to the graph of the function $y = f(x)$ over the interval [a,e]



A. List the point(s) on the *x*-axis where $f'(x) = 0$ ______________________________

B. List the point(s) on the *x*-axis where $f''(x) = 0$ ______________________________

- Express intervals in the form $a < x < b$ or (a, b) your preference of notation.

C. Find all intervals on which $f'(x) < 0$ ______________________________

D. Find all intervals on which $f''(x) > 0$ ______________________________

(4)

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4. Given $s = \ln t$ Evaluate $\left.\frac{ds}{dt}\right|_{t=8}$

4.______________________________(2)

5. Given $y = f(x) = 3^x$ Approximate to 4 decimal places $f'(2)$.

5.______________________________(2)

6. Given $y = f(x) = e^{10x}$ Determine $f''(x)$

6.______________________________(2)

7. Let $y = \frac{-1}{2x} + 10\sqrt{x} - 3$. Compute $\frac{dy}{dx}$.

7.______________________________(2)

8. Find the derivative of the function $y = x^2 \cdot \ln x$

8.______________________________(2)

9. Find the derivative of the function $w = (5x^2 + 1)^{10}$

9.______________________________(2)

10. Find an equation of the tangent line to the curve $y = f(x) = x^4 + 1$ at $x = 1$.

10.______________________________(2)

11. Find the quantity q which maximizes profit if the total revenue and total cost (in dollars) are given by

$$R(q) = 420q$$
$$C(q) = 10{,}500 + 5q^2$$

11.____________________________(2)

12. A state park charges $200 for an annual pass. At this rate 715 people purchase passes every year. For each $10 decrease in price 65 more people purchase a pass. What price should the park charge in order to maximize revenue?

12.___________________________(3)

13. Consider a function defined over the entire real line such that $f'(x) = 4x + 6$

(a) When (over what interval) is f increasing? 13(a)______________________(1)

(b) When (over what interval) is f decreasing? 13(b)______________________(1)

14. (bonus) A car moves with velocity $v(t) = \frac{60}{(50)^t}$ miles per hour where t is the time in hours. Set up, but **do not evaluate**, a definite integral for the distance traveled in the first hour.

14.________________________(1)

15. Find the indefinite integral $\int \frac{-1}{3x^5}\, dx$

15.________________________(2)

16. Oil is leaking out of a tanker at a rate of $r(t) = 80e^{(-0.02)t}$ gallons per minute where t is the elapsed time in minutes. How much leaks out during the first hour?

16. __________________________(3)

17. Evaluate $\int_4^{25} \sqrt{x}\, dx$. Simplify your answer

17.__________________________(3)

18. Using a definite integral find the area of the region below the curve and above the *x*-axis for the inverted parabola: $y = f(x) = -x^2 + 2x$.

18.____________________________(3)

19. The marginal cost of a product is $C'(q) = q^2 - 50q + 700$ dollars. The fixed costs are 500. What is the total cost to produce 30 items?

19.____________________________(2)

20. Consider the polynomial $y = f(x) = x^3 - 3x^2$ **restricted** to the interval $\left[-\frac{1}{2}, 4\right]$

For your convenience: $f'(x) = 3x^2 - 6x$ and $f''(x) = 6x - 6$

(a) Find any critical points (*Make sure you find both 1st and 2nd coordinate for these critical points*)

(2)

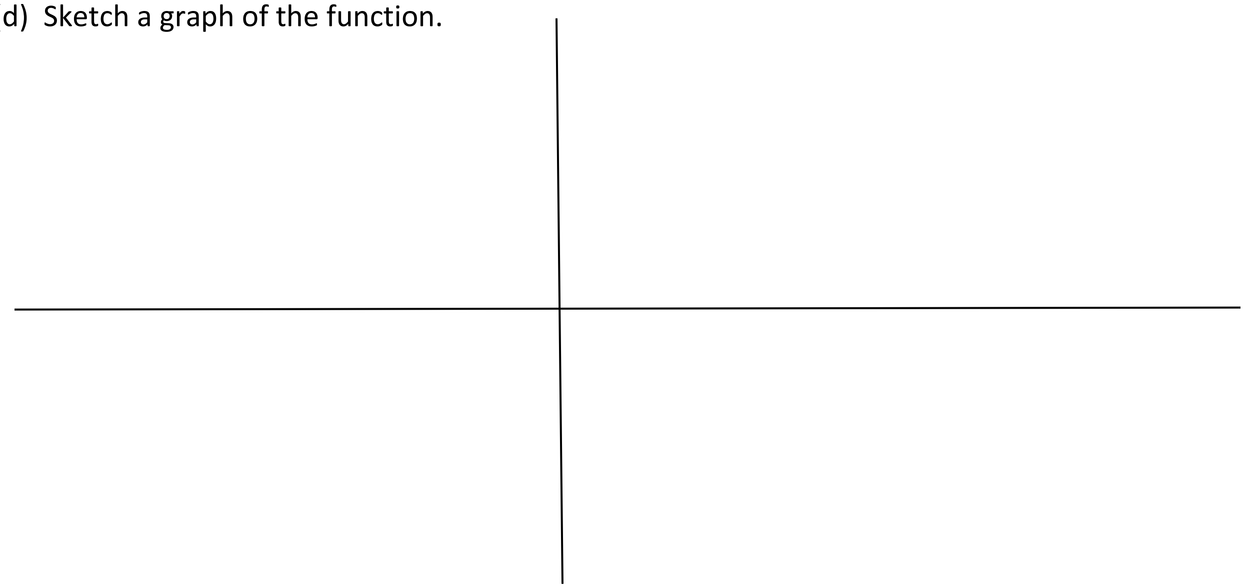
(b) Use the 1st or 2nd derivative test to classify these critical points as local max or local min

(2)

(c) Find any global max or global min

(2)

(d) Sketch a graph of the function.



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