MATH 119 **Sample (B) Key** ***(Chapter 2 and 3)*** NAME:
Class ID #:

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1) The population of a town in millions is given by: $P = 1.2(1.01)^t$ where t is the number of years since the start of 1998 (i.e. $t = 0$ corresponds the year 1998). Find:

a) The population in 2000 ***P = 1.22412 B***

b) The average rate of growth between 1998 and 2000:

0.01206 M/Y

c) How fast the population is growing at the start of 1998? (Hint: Estimate the instantaneous rate of change of P at $t = 0$ using $h = 0.01$)

0.01194045 M/Y

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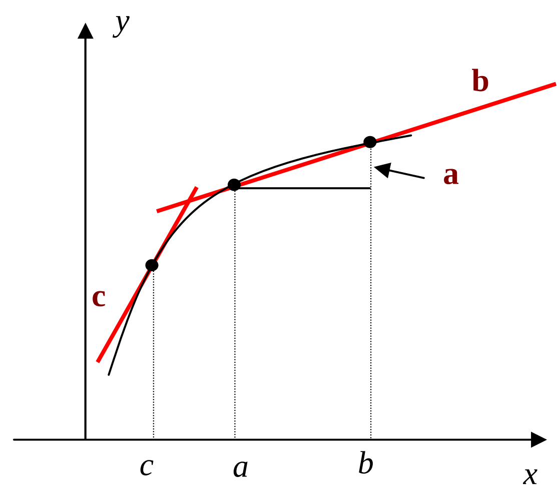
2) Use the graph on the right to sketch following:

a) the line segment corresponding to $f(b) - f(a)$; label that line segment as **<u>line *A*</u>**;

b) the line whose slope is given by $\dfrac{f(b) - f(a)}{b - a}$; label that line as **<u>line *B*</u>**;

c) the line whose slope is given by $f'(c)$; label that line as **<u>line *C*</u>**;



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3) The distance s traveled by an object as a function of time t is given in the following table:

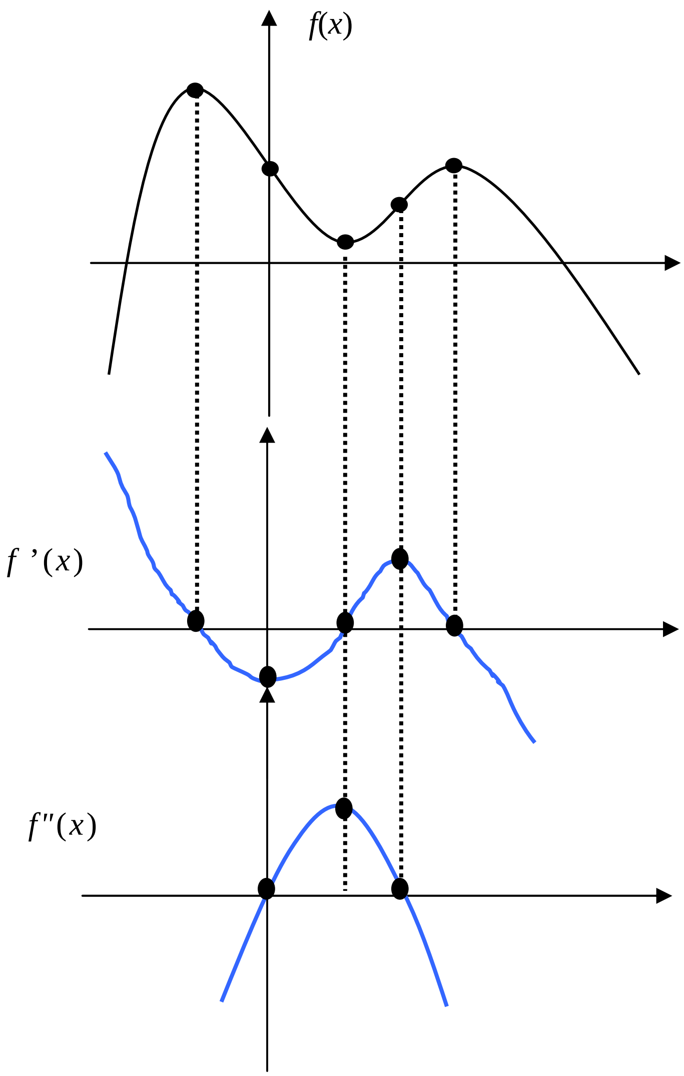
t (sec)	**0**	**1**	**2**	**3**	**4**	**5**
s (feet)	0	2	5	9	15	27

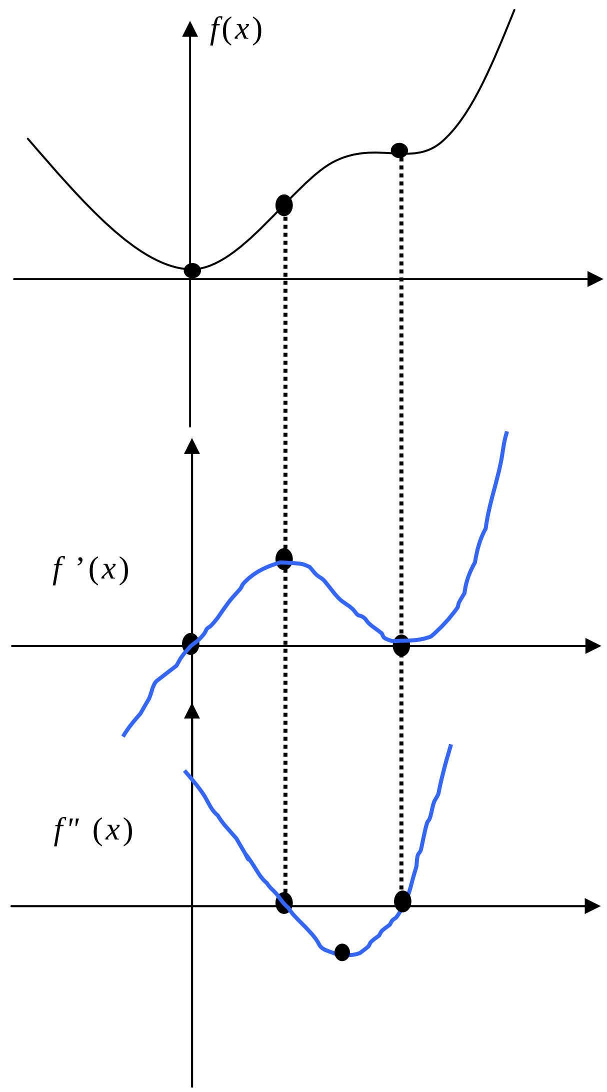
a) Find the average velocity of the object between $t = 1$ and 4.

Av = 4.3

b) Estimate the velocity of the object at $t = 3$. (you can use one interval only) ***V = 6***

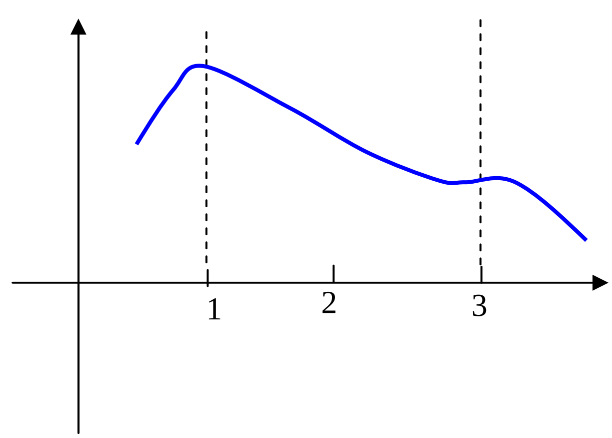
4) Sketch the graph of the first and second derivatives of the functions given below. Be sure that your sketches are consistent with the important features of the original functions.





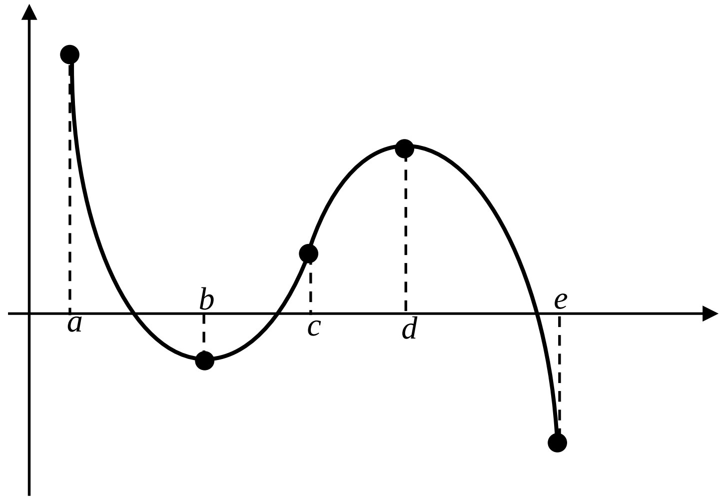
5) Draw a possible graph of $y = f(x)$ given the following information about its derivative:

- $f'(x) > 0$ for $x < 1$
- $f'(x) < 0$ for $1 < x < 3$ and $x > 3$
- $f'(x) = 0$ at $x = 1$ and $x = 3$



6) Using the following graph, estimate the *intervals* or *points* where:

$f'(x) > 0$	$b < x < d$
$f'(x) < 0$	$a < x < b$ $d < x < e$
$f''(x) = 0$	$x = c$
$f''(x) > 0$	$a < x < c$
$f''(x) < 0$	$c < x < e$
$f''(x) < 0$ and $f'(x) = 0$	$x = d$
$f''(x) > 0$ and $f'(x) = 0$	$x = b$



7) Suppose that $f(t)$ is a function, that $f(10) = 7$ and that $f'(10) = -0.2$. Use this information to estimate $f(12)$.

f(12) = 6.6

8) If $f(x) = x^2 + 3x$, find $f'(2)$. (use $h = 0.01$, and show all steps)

When h = 2.01,	***f ' (2) = 7.01***
When h = 1.99,	***f ' (2) = 6.99***
Then	***f ' (2) = 7***

9) Find an equation of the tangent line to the graph of $y = e^{-2x} + 2$ at $x = 0$

y = -2x + 3

10) Find an equation of the tangent line to the graph of $y = 3x^2 - 4x + 3$ at $x = 1$

y = 2x

11) The population in a state is increasing about 3% a year. If t is time in years since 1990, the population P in millions can be approximated by: $P = 10(1.03)^t$

a) What is the population will be in the year 2000?

P = 13.439

b) How fast (in people/year) does this model predict the population will be increasing in the year 2000?

P ' = 0.397

12) Assume the demand function for a certain product is: $q = 1000e^{-0.02p}$ (p is the price, q is the quantity)

a) Write the revenue R as a function of price

***R* = (price per unit)(number of units) or** $R = 1000p \cdot e^{-0.02p}$

b) Find the rate of change of revenue R' with respect to price ?

$$R' = 1000e^{-0.02p} - 20p \cdot e^{-0.02p}$$

c) find $R(10)$ and $R'(10)$

***R* (10) = $8187.31**
***R* ' (10) = $654.98**

13) Using any method you wish, find $\frac{dy}{dx}$ for each of the following (* write your answer with positive exponent for *a* and *c*):

*a) $y = \frac{4}{x^2} + 6\sqrt[4]{x} - 8x$

$$y' = \frac{-8}{x^3} + \frac{3}{2x^{3/4}} - 8$$

b) $y = e^{-4x} - 4x^2$

$$y' = -4e^{-4x} - 8x$$

*c) $y = 8 \cdot \sqrt{x^4 - 2}$

$$y' = \frac{16x^3}{\sqrt{(x^4 - 2)}}$$

d) $y = \ln(x^2 - 2) + 3\sqrt{x}$

$$y' = \frac{2x}{x^2 - 2} + \frac{3}{2\sqrt{x}}$$

e) $y = (e^{2x} - 4)^3$

$$y' = 6e^{2x}(e^{2x} - 4)^2$$

f) $y = 4.(5^x) + 5(2^x)$

$$y' = 4.\ln 5.5^x + 5.\ln 2.2^x$$

* g) $y = e^{\sqrt{4x-2}}$

$$y' = \frac{2.e^{\sqrt{4x-2}}}{\sqrt{4x-2}}$$

h) $y = (2 + \ln 6x)^4$

$$y' = \frac{4(2 + \ln 6x)^3}{x}$$

14) Using any method you wish, find $\frac{dy}{dx}$ for each of the following:

a) $y = (2x^3 + 4).\ln 5x$

$$y' = 6x^2 \ln 5x + \frac{(2x^3 + 4)}{x}$$

b) $y = 5x^4.e^{4x}$

$$y' = 20x^3e^{4x} + 20x^4e^{4x}$$