MATH 119 **Sample (A Key)** ***(Chapter 2 and 3)*** NAME:

Class ID #:

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1) The population of a town in millions is given by: $P = 1.5(1.05)^t$ where t is the number of years since the start of 1995. Find:

a) The average rate of growth between 1995 and 2000:

0.0828 M/Year

b) How fast the population is growing at the start of 1995? *(hint: find the instantaneous rate of change, the derivative at t = 0 for 1995)*

0.073 M/Year

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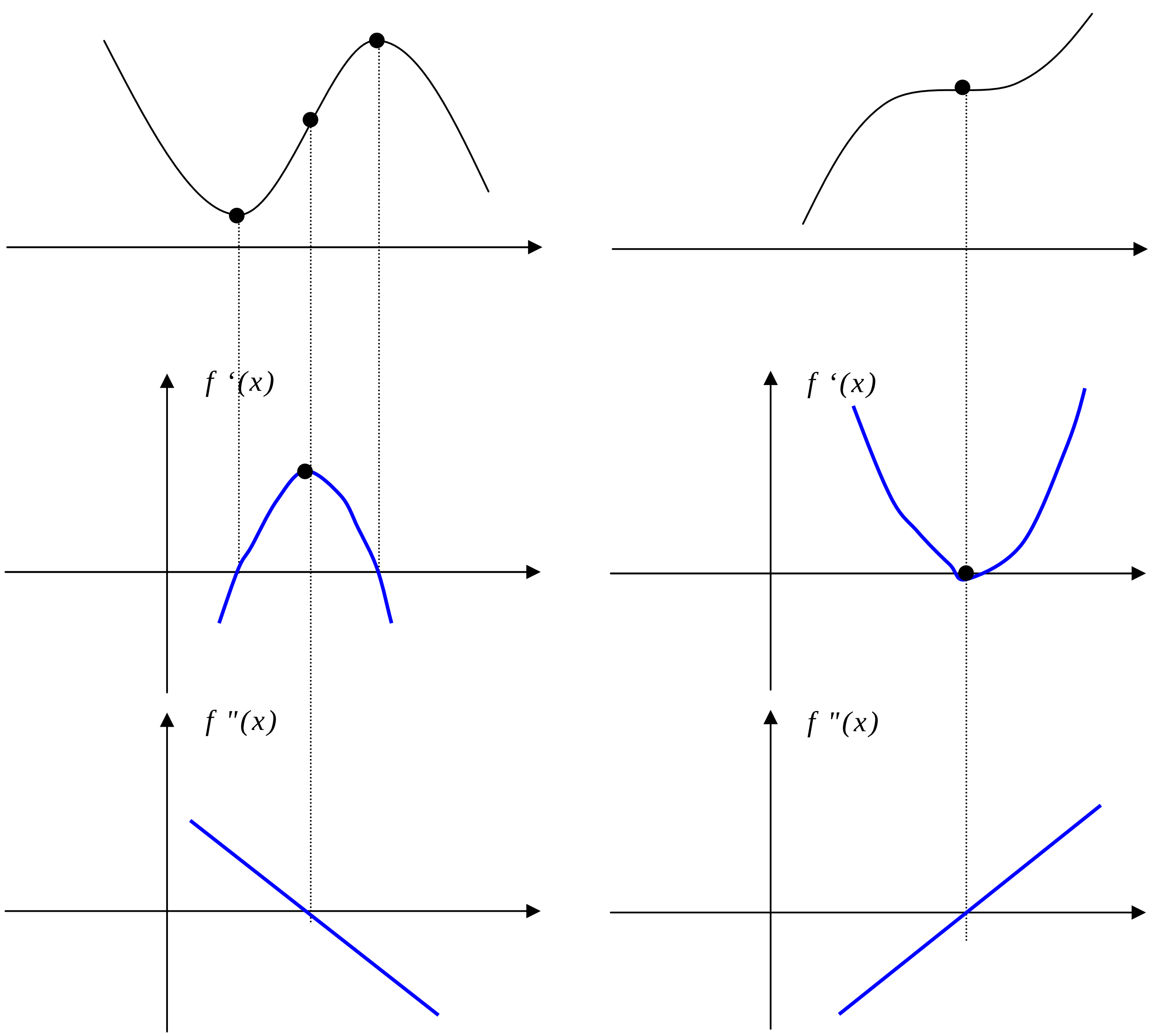
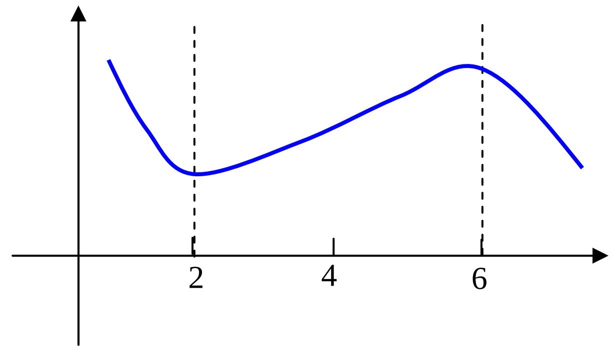
2) Sketch the graph of the first and second derivatives of the functions given below. Be sure that your sketches are consistent with the important features of the original function.



3) Draw a possible graph of $y = f(x)$ given the following information about its derivative:

- $f'(x) > 0$ on $2 < x < 6$
- $f'(x) = 0$ at $x = 2$ and $x = 6$
- $f'(x) < 0$ on $x < 2$ and $x > 6$



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4) Let $C(q)$ represent the total cost of producing q items. Suppose that $C(10) = \$15$ and $C'(10) = 0.2$. Find the total cost of producing 8 items.

$14.6

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5) If $f(x) = x^2 - 2x$, find $f'(2)$. (use $h = 0.001$ and show all steps)

When h = 2.001,	***f ' (2) = 2.001***
When h = 1.999,	***f ' (2) = 1.999***
Then	***f ' (2) = 2***

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6) A company's revenue R is a function of advertising expenditure, a. Suppose $R = f(a)$

a) What does $f'(100) = 0.8$ mean?

Every additional $1 in advertisement, there is $0.8 more in revenue.

b) If $f'(100) = 0.8$, should the company spend more or less in advertising? Why?

No because for every $1 spent, there $0.8 in revenue or -$0.2 in retu

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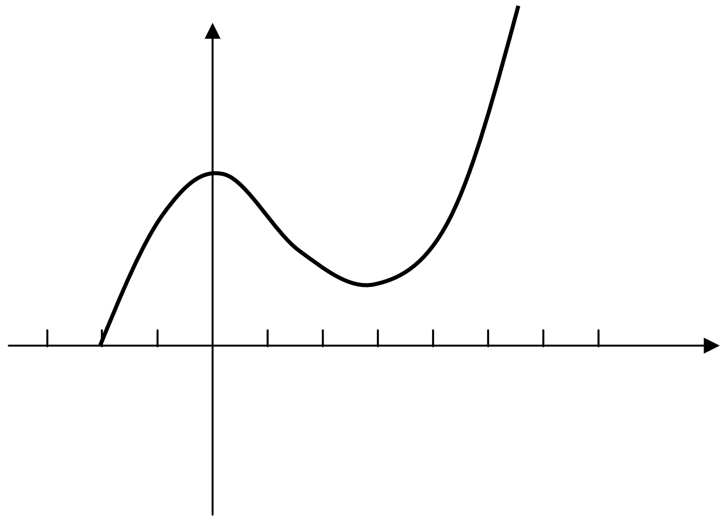
7) Suppose that $f(t)$ is a function, that $f(10) = 5$ and that $f'(10) = -0.1$. Use this information to estimate $f(15)$.

f(15) = 4.5

8) Using the following graph, estimate the *intervals* or *points* where:

$f'(x) > 0$	***-2 < x < 0*** ***x > 3***
$f'(x) < 0$	***0 < x < 3***
$f''(x) = 0$	***x = 1***
$f''(x) > 0$	***1 < x***
$f''(x) < 0$	***x < 1***



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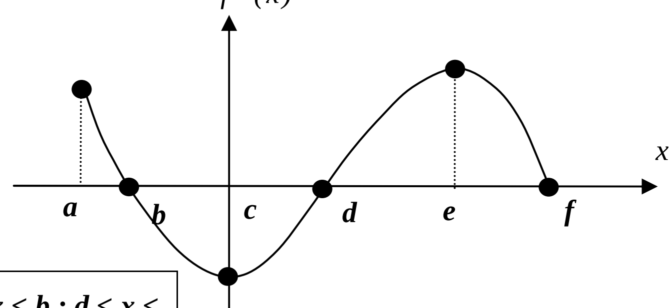
9) Referring to the graph of a **<u>derivate</u>** $f'(x)$ of a function $f(x)$ for $a \le x \le f$ given below, indicate the points or intervals where the following conditions can hold.



The **Original** function is Increasing in:	***a < x < b ; d < x < f***
The **Original** function is Decreasing in:	***b < x < d***
The **Original** function has Inflection Points at:	***x = c , x = e***
The **Original** function has Horizontal Tangent line at:	***x = b, x = d, x = f***

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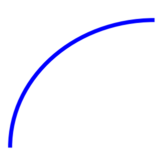
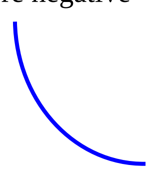
10) Draw a possible graph of a function whose:

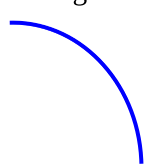
a) Second derivative is everywhere negative but first derivative is everywhere positive



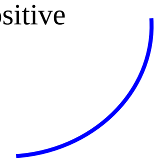
b) Second derivative is everywhere positive but first derivative is everywhere negative



c) Second derivative is everywhere negative and first derivative is everywhere negative



d) Second derivative is everywhere positive and first derivative is everywhere positive



11) Find the points on the graph of $y = x^2 - 4x + 10$ where the tangent line is horizontal

Horizontal or solve for y ' = 0, then x = 2 and y = 6 and the point is (2 , 6)

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12) Find an equation of the tangent line to the graph of $y = e^{-4x} + 4$ at $x = 0$

y = -4x + 5

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13) Find an equation of the tangent line to the graph of $y = 2x^2 - 4x + 3$ at $x = 1$

y = 1

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14) Find an equation of the tangent line to the graph of $y = 2^x$ at $x = 1$

y = 1.386x + 0.6137

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15) The population in Hungary is decreasing, currently about 10% a year. If t is time in years since 1990, the population P in millions can be approximated by: $P = 10(0.9)^t$

a) What is the population will be in the year 2000?

P = 3.49

b) How fast (in people/year) does this model predict the population will be decreasing in the year 2000?

P ' = -0.3674 / year   or 36.74%

16) Using any method you wish, find $\frac{dy}{dx}$ for each of the following (* write your answer with positive exponent for *a* and *c*):

a) $y = \frac{4}{x^3} + 4\sqrt[3]{x} - 4x + 2$

$$y' = \frac{-12}{x^4} + \frac{4}{3x^{2/3}} - 4$$

b) $y = 4(x^3 + 1)^5$

$$y' = 60x^2(x^3 + 1)^4$$

c) $y = \sqrt{x^6 - 4}$

$$y' = \frac{3x^5}{\sqrt{(x^6 - 4)}}$$

d) $y = \ln(x^2 - 1)$

$$y' = \frac{2x}{x^2 - 1}$$

e) $y = (e^{2x} - 4)^3$

$$y' = 6e^{2x}(e^{2x} - 4)^2$$

f) $y = e^{-4x} + 5x$

$$y' = -4e^{-4x} + 5$$

g) $y = 5.(4^x)$

$$y' = 5.\ln 4.4^x$$

h) $y = (2 + \ln 5x)^3$

$$y' = \frac{3(2 + \ln 5x)^2}{x}$$

7) Using any method you wish, find $\frac{dy}{dt}$ for each of the following

i) $y = (t^4 + 4).e^{2t}$

$$y' = 4t^3 e^{2t} + 2e^{2t}(t^4 + 4)$$

k) $y = t^4.\ln 6t$

$$y' = 4x^3 \ln 6x + x^3$$