DEPARTMENTAL FINAL EXAMINATION
Fall 2008

MATH M 119

BRIEF SURVEY OF CALCULUS

Directions

- **DO NOT OPEN** this test booklet until you are told to do so.
- There are 8 pages in this exam with 17 problems - You MUST get a new exam from the proctor if your exam is incomplete.
- PRINT your name, student ID#, and mark your section below.
- You will have two hours to complete this examination.

No scrap paper, notes, books, nor graphing calculators allowed. **Cell phones should be OFF. Headsets are not permitted.**

Please write legibly. To receive any credit you must show supporting work!

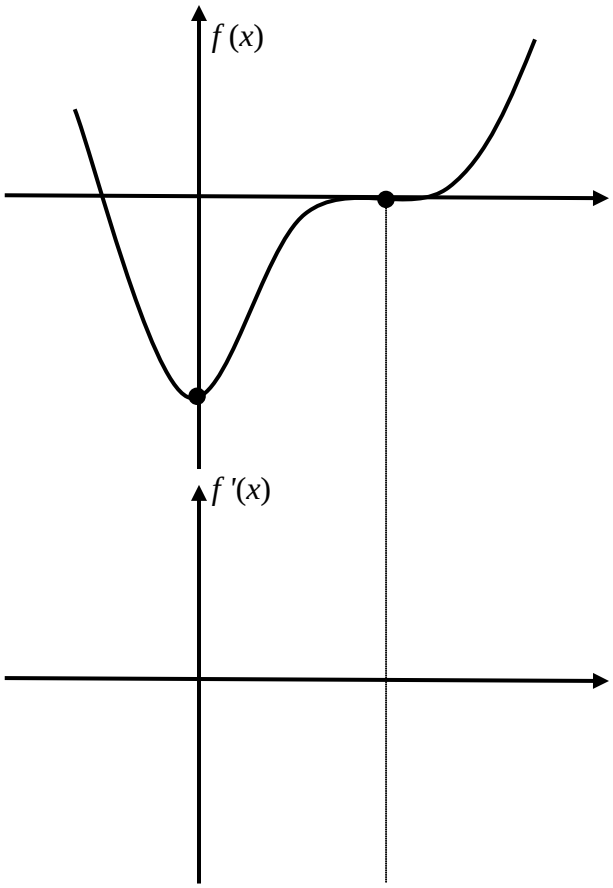
Circle All Your Final Answers

NAME (Print Clearly)	
STUDENT ID#	

Do Not Write in this area.		
1	**3p**	
2	**4p**	
3	**6p**	
4	**3p**	
5	**4p**	
6	**4p**	
7	**16p**	
8	**6p**	
9	**4p**	
10	**6p**	
11	**16p**	
12	**6p**	
13	**4p**	
14	**4p**	
15	**2p**	
16	**4p**	
17	**8p**	
Total	**100p**	

Check your section here.

1. Sketch the graphs of the first derivative of the function given below. Be sure that your sketches are consistent with the important features of the original functions. **[3p]**



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2. A company's pricing schedule is given as: *(Show all steps for credit)*

q (number of units)	1	3	5	7
p (price per unit)	12	10	8	6

Find a linear equation which expresses p as a function of q **[4p]**

3. Referring to the graph of the function $f(x)$ in $a \le x \le e$ given below **[6p]**

- List the points on *x*-axis where:

A) $f'(x) = 0$: _____

B) $f''(x) = 0$: _____

- Express intervals in the form $a < x < b$ or $(a\ ,\ b)$ your choice of notation.

C) Find all intervals on which $f'(x) > 0$:

_____

D) Find all intervals on which $f'(x) < 0$:

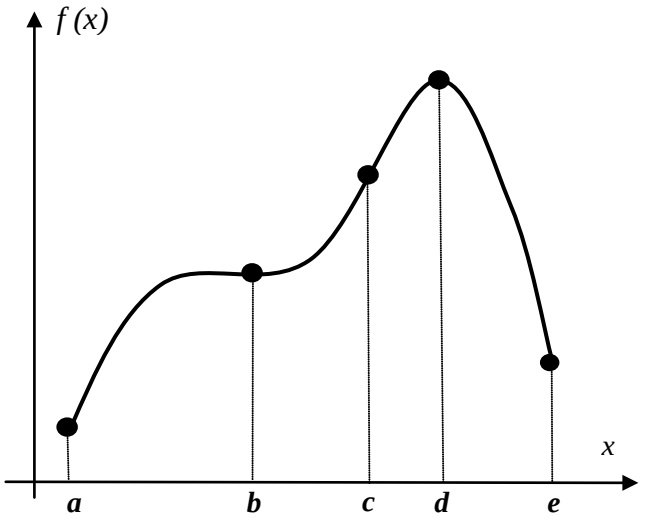
_____

E) Find all intervals on which $f''(x) > 0$:

_____

F) Find all intervals on which $f''(x) < 0$:

_____



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4. A population, currently 1200, grows at a continuous rate of 2 % per year. What will the population be in 8 years? **[3p]**

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5. The amount $P(t)$ of a radioactive substance remaining after t days may be expressed by the formula **[4p]** $P(t) = P_{\circ}e^{kt}$ where $P_{\circ}$ is the initial quantity. If the half-life of the substance is 8 days, determine the decay rate k (*write your answer with 4 decimal places*)

6. Suppose that $f(x)$ is a function with $f(10) = 200$ and $f'(x) = 3$. Estimate $f(12)$. **[4p]**

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7. Using any method you wish find $\frac{dy}{dx}$ for each of the following: *(Show all steps for credit)* **[4p each]**

(a) $y = 0.08x^2 - \frac{2}{3}x + \frac{5}{x^2}$

(b) $y = 5^x$

(c) $y = e^{(x^2+1)}$

(d) $y = x^2 \cdot \ln x$

8. Find an equation of the tangent line to the graph of $y = x^3$ at $x = 2$. **[6p]**

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9. Revenue is given by $R(q) = 300q$ and cost is given by $C(q) = 1000 + 2q^2$. How many units must be sold in order to maximize the profit? **[4p]**

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10. Find the present value of an investment over 8 years period if there is a continuous money stream of \$5000 per year and the current interest rate is 4% compounded continuously. **[6p]**

11. Evaluate each of the following integrals: *(Show all steps for credit)* **[4p each]**

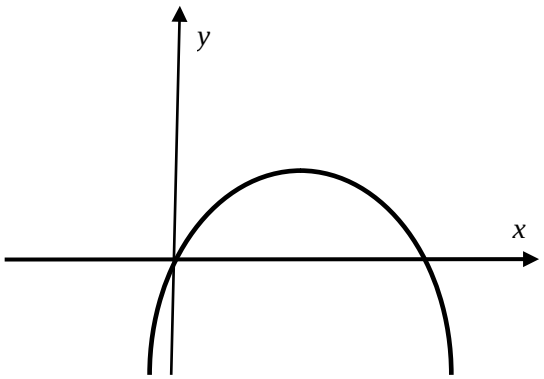
(a) $\int \sqrt{x}\, dx$

(b) $\int (e^{5x} - \frac{1}{x^3})\, dx$

(c) $\int_{-1}^{3} (4x - 1) dx$

(d) $\int_{1}^{5} \frac{1}{x}\, dx$

12. Find its area using the definite integral bounded by the graph of $y = -x^2 + 5x$ and the x-axis **[6p]**



13. The marginal cost for a company is given by $C'(q) = 3q^2 - 6q + 20$ dollars/unit where q is the quantity produced. If $C(0) = 100$, find the total cost of producing 10 units. **[4p]**

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14. Find an antiderivative $F(x)$ with $F'(x) = e^{4x}$ and $F(0) = 2$. **[4p]**

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15. Oil is leaking out of a tanker at a rate of $r(t)$ gallons per minute where t is the elapsed time in minutes. Write a definite integral expressing how much oil leaks out in the first three hours. (*You are not required to estimate nor evaluate this integral*) **[2p]**

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16. Consider a function $f(x)$ defined over the entire real line such as $f'(x) = 2x + 10$ **[4p]**

(a) When (over what interval) is f is increasing? 16(a) _____

(b) When (over what interval) is f is decreasing? 16(a) _____

17. For the function given by: $f(x) = x^3 + 3x^2$ **[8p]**

(a) Find the critical points and determine whether any of these are local maximum or local minimum
(b) Use the second derivative to find the inflection points, the intervals for concave up, concave down

Show all steps and put your final answers for part *a* and *b* in the tables below to be graded. **Final answers without work will not be graded**.

For your convenience: $f'(x) = 3x^2 + 6x$ *and* $f''(x) = 6x + 6$

Local Max at (*x* coord.)	
Max value (*y* coord.):	
Local Min at (*x* coord.)	
Min value (*y* coord.):	

Inflection points at:	
Concave Up on interval(s):	
Concave Down on interval(s):	

Combine the above information with the behavior of the function at $\pm\infty$ to sketch a graph of $f(x)$.