MATH 119	Chapter 3: Derivative Formulas

Derivative Rules	**Examples**
1. The main Rules: If $y = x^a$; then $y' = ax^{a-1}$ If $y = x$; then $y' = 1$ If $y = c$; then $y' = 0$	Ex: $y = x^{-2} - x + 2$ $y' = -2x^{-3} - 1$ Ex: $y = 3x^4 - 5x + 4$ $y' = 12x^3 - 5$
2. The ln rule If $y = \ln f(x)$; then $y' = \frac{f'(x)}{f(x)}$	Ex: $y = \ln(5x-4)$ $y' = \frac{5}{5x-4}$
3. The *e* rule: If $y = e^{f(x)}$; then $y' = f'(x) \cdot e^{f(x)}$ y' = (derivative of the power). the original	Ex: $y = 4e^{-5x}$ $y' = 4(-5)e^{-5x} = -20e^{-5x}$
4. The *a* rule: If $y = a^x$; then $y' = \ln a . a^x$	Ex: $y = 4 \cdot 5^x$ $y' = 4 \cdot \ln 5 \cdot 5^x$
5. The chain (power) rule: If $y = \left(f(x)\right)^n$; then $y' = n\left(f(x)\right)^{n-1} . f'(x)$ y' = (derivative of outisde). (derivative of inside)	Ex: $y = (5-3x)^9$ $y' = 9(5-3x)^8.(-3)$ $= -27(5-3x)^8$
6. The mutiplication rule: $y = f(x).g(x)$; then $y' = f'(x).g(x) + g'(x).f(x)$ or, it is easier this way: If y = (first)(second) ; then: y' = (derivative of first)(second) + (derivative of second)(first)	Ex: $y = (3x-2)(5-6x)$ $y' = 3(5-6x) + (-6)(3x-2)$ $= 15 - 18x - 18x + 12$ $= 27 - 36x$
7. The quotient rule: $y = \frac{f(x)}{g(x)}$; then $y' = \frac{f'(x).g(x) - g'(x).f(x)}{[g(x)]^2}$ or, it is easier this way: If $y = \frac{\text{Num}}{\text{Den}}$; then: $y' = \frac{\text{(derivative of Num)(Den) - (derivative of Den)(Num)}}{\text{Den}^2}$	Ex: $y = \frac{2x-3}{5x+1}$ $y' = \frac{(2)(5x+1) - (5)(2x-3)}{(5x+1)^2}$ $= \frac{10x + 2 - 10x + 15}{(5x+1)^2}$ $= 17x \big/ (5x+1)^2$

- Rules 1, 2, 3 and 4 are from sections 3.1 and 3.2.
- Rule 5 is from section 3.3.
- Rules 6 and 7 are from section 3.4.

Finding the Equation of the Tangent Line: *This part is available as a power point presentation*

Example 1: Find the equation of the tangent line of : $y = x^3 - x^2$ at $x = 2$

Solution:

The slope of the tangent line at any point is the derivative at that point:

$$y' = 3x^2 - 2x$$

At $x = 2$, we need to find m, and the value of y

at $x = 2$, $y' = 8$ or $m = 8$

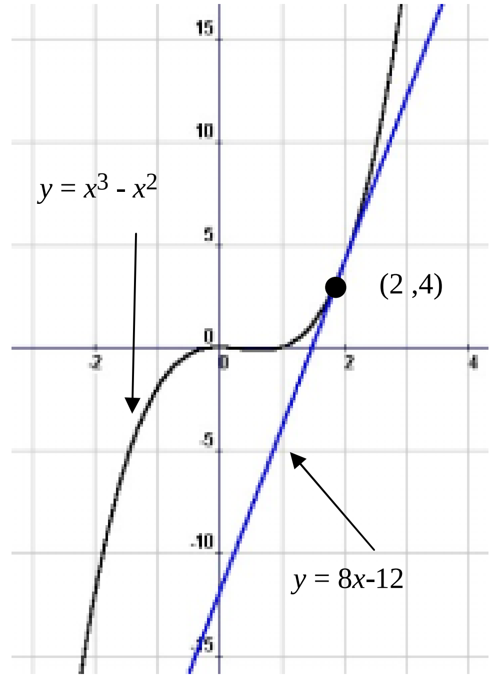
at $x = 2$, $y = 4$

Now, we want to find the equation of the line that passes the point (2 , 4) with $m = 8$

$y = mx + b$

$4 = 8(2) + b$ then $b = -12$

The answer : $y = 8x - 12$



Example 2: Find the equation of the tangent line of : $y = x^3 + x^2 - x + 2$ at $x = 1$

Solution:

The slope of the tangent line at any point is the derivative at that point:

$$y' = 3x^2 + 2x - 1$$

At $x = 1$, we need to find m, and the value of y

at $x = 1$, $y' = 4$ or $m = 4$

at $x = 1$, $y = 3$

Now, we want to find the equation of the line that passes the point (1 , 3) with $m = 4$

$y = mx + b$

$3 = 4(1) + b$ then $b = -1$

The answer : $y = 4x - 1$

Example 3: Find the points where the tangent line is horizontal for :

$y = x^3 - 3x + 4$.

Solution:

If the tangent line is <u>horizontal</u>, then the slope $m = 0$ or $y' = 0$:

$$y' = 3x^2 - 3$$

$$= 3(x^2 - 1) = 3(x - 1)(x + 1)$$

Make $y' = 0$ and solve: $x = 1$ and $x = -1$

The points where the tangent line is horizontal are:

(1 , 2) and (-1 , 6)