Compound Interest: $$P = P_\circ\left(1+\frac{r}{n}\right)^{nt}$$

P_0: the principal, amount invested P: the new balance t: the time in years
r: the rate, (in decimal form) n: the number of times it is compounded.

Ex1: Suppose that \$5000 is deposited in a saving account at the rate of 6% per year. Find the total amount on deposit at the end of 4 years if the interest is:

P_0 =\$5000, r = 6% , t = 4 years

a) compounded annually, n = 1: $P = 5000(1 + 0.06/1)^{(1)(4)} = 5000(1.06)^{(4)} = \$6312.38$

b) compounded semiannually, n =2: $P = 5000(1 + 0.06/2)^{(2)(4)} = 5000(1.03)^{(8)} = \$6333.85$

c) compounded quarterly, n = 4: $P = 5000(1 + 0.06/4)^{(4)(4)} = 5000(1.015)^{(16)} = \$6344.93$

d) compounded monthly, n =12: $P = 5000(1 + 0.06/12)^{(12)(4)} = 5000(1.005)^{(48)} = \$6352.44$

e) compounded daily, n =365: $P = 5000(1 + 0.06/365)^{(365)(4)} = 5000(1.00016)^{(1460)} = \$6356.12$

For Compounded Annually where *n* =1: $$P = P_\circ\left(1+\frac{r}{n}\right)^{nt} = P_\circ(1+r)^t$$

Continuous Compound Interest:

Continuous compounding means compound every instant, consider investment of 1\$ for 1 year at 100% interest rate. If the interest rate is compounded n times per year, the compounded amount as we saw before is given by:

$$P = P_0\,(1+ r/n)^{nt}$$

The following table shows the compound interest that results as the number of compounding periods increases:

P_0 = \$1; r = 100% = 1; t = 1 year

Compounded	n	Compound amount
annually	1	$(1+1/1)^1$ = \$2
monthly	12	$(1+1/12)^{12}$ = \$2.6130
daily	360	$(1+1/360)^{360}$ = \$2.7145
hourly	8640	$(1+1/8640)^{8640}$ = \$2.71812
each minute	518,400	$(1+1/518{,}400)^{518{,}400}$= \$2.71827

As the table shows, as n increases in size, the limiting value of P is the special number **e = 2.71828**

If the interest is compounded continuously for t years at a rate of r per year, then the compounded amount is given by:

$$P = P_0 . e^{rt}$$

Ex2: Suppose that \$5000 is deposited in a saving account at the rate of 6% per year. Find the total amount on deposit at the end of 4 years if the interest is compounded continuously. *(compare the result with example 1)*

P_0 =\$5000, r = 6% , t = 4 years

$P = 5000.e^{(0.06)(4)} = 5000.(1.27125) = \$6356.24$

Ex3: Find the amount to be invested at a rate of 8% compounded continuously in order to get \$12,930 in 6 years.

$P = \$12{,}930, \quad r = 0.08, \quad t = 6$ years.

$\$12{,}930 = P_0 . e^{(0.08)(6)}$, then $P_0 = \$8000$

The Growth, Decline:

Continuous Growth: $\rightarrow P = P_o . e^{rt}$	***Continuous Decline:*** $\rightarrow P = P_o . e^{-rt}$
Annual Growth: $\rightarrow P = P_o (1+r)^t = P_o a^t$ *where* $a > 1$	***Annual Decline:*** $\rightarrow P = P_o (1-r)^t = P_o b^t$ *where* $0 < b < 1$

Doubling Time = $\dfrac{\ln 2}{r}$ *(for continuous growth)* $= \dfrac{\ln 2}{\ln(1+r)}$ *(for annual growth)*

Half-life = $\dfrac{\ln 2}{r}$

Ex4: The growth rate in a certain country is 15% per year. Assuming continuous growth :

a) If the population is 100,000 now, find the new population in 5 years.
b) When will the 100,000 double itself?

Ex5: In 1965 the price of a math book was \$16. In 1980 it was \$40. Assuming the continuous growth :

a) Find r and write the equation.
b) Find the cost of the book in 1985.
c) After when will the cost of the book be \$32 ?

Ex6: A couple want an initial balance to grow to \$ 211,700 in 5 years. The interest rate is compounded continuously at 15%. What should be the initial balance?

Ex7: The population of a city was 250,000 in 1970 and 200,000 in 1980 *(Decline)*. Assuming the population is decreasing continuously, find the population in 1990.

Ex8: At what rate compounded annually will an investment of \$2100 accumulate to \$3400 by the end of 6 years

Ex9: How long does it take amount to double at 8.5% compounded: annually, continuously?

Ex10. The population of a certain town is declining exponentially due to immigration. If the population was reduced by 20% after 10 years, find the decline rate.

Ex11. Write the equatiion of problem 10 but: If only 85% the population are present after 10 years.

Ex12.. The half-life of a certain radioactive substance is 12 days. If there are 10 grams initially:

a) find the rate.
b) when will the substance be reduced to 2 grams?

Ex13. Convert the function $P = 400e^{0.05t}$ to the form $P = P_{\circ} a^t$

Ex14. Convert the function $P = 2000(1.08)^t$ to the form $P = P_{\circ} e^t$

Answers:	**4.** (211700, 4.62)	**5.**(6.1%, 54.28, 11.36)	**6.** (100,000)	**7.** (160,000)
	8. (8.36%)	**9.** (8.5, 8.15)	**10.** (2.23%)	**11.** $0.85 = e^{-10t}$
	12. (5.78% , 27.86)	**13.** $P = 400(1.0513)^t$	**14.** $P = 2000e^{0.07696\,t}$	