

Math M119

Logarithms and Functions Review

$$y = \log_a x \Rightarrow a^y = x$$

The number a is called the *logarithmic base*

If $a = 10$, then it is \log_{10} and it is called **Common logarithm** (available in calculator as \log)

If $a = e$, then it is \log_e or \ln and it is called **Natural logarithm** (available in calculator as \ln)

$$\log_{10} x = \log x \text{ (Common Logarithm)}$$

$$\log_e x = \ln x \text{ (Natural Logarithm)}$$

1. Convert the following from exponential form to logarithmic form:

a) $y = x^2$

Answer: $\log_x y = 2$

b) $2^3 = 8$

Answer: $\log_2 8 = 3$

c) $9^{1/2} = 3$

Answer: $\log_9 3 = 1/2$

d) $e^2 = x$

Answer: $\log_e x = 2$ or $\ln x = 2$ (base of e means \ln)

e) $10^2 = 100$

Answer: $\log_{10} 100 = 2$ or $\log 100 = 2$ (no base means base of 10)

2. Convert the following from logarithmic to exponential form:

a) $\log_x 9 = 3$

Answer: $x^3 = 9$

b) $\log x = -1$

Answer: $10^{-1} = x$ (no base means base of 10)

c) $\ln x = -1$

Answer: $e^{-1} = x$ (\ln means log to the base of e)

3. Solve for x (hint: first, convert each from logarithmic to exponential form)

a) $\log_9 x = 1$

Answer: $x = 9$

b) $\log_a x = 1$

Answer: $x = a$

c) $\ln x = 1$

Answer: $x = e$ (why?)

4. Solve for x (hint: first, convert each from logarithmic to exponential form)

a) $\log_9 x = 0$

Answer: $x = 1$

b) $\log_x x = 0$

Answer: $x = 1$

c) $\ln x = 0$

Answer: $x = 1$ (why?)

$$\log_a a = 1; \log_e e = 1 \text{ or } \ln e = 1$$

$$\log_a 1 = 0; \log_e 1 = 0 \text{ or } \ln 1 = 0$$

Properties of logarithms

Rule	Formula	Example
I) Multiplication	$\ln(AB) = \ln A + \ln B$	$\ln 5x = \ln 5 + \ln x$
II) Division	$\ln \frac{A}{B} = \ln A - \ln B$	$\ln 5/x = \ln 5 - \ln x$
III) Power	$\ln A^p = p \ln A$	$\ln 5^x = x \ln 5$

Examples:

Example 1 : Express in term of logarithms:

a) $\log(x^2y^2)$

b) $\log \frac{x^3y^2}{z^4}$

c) $\log \frac{\sqrt{x^3y^2}}{z^3w^5}$

Example 2: Express as a single logarithm:

a) $3 \ln x + 4 \ln y - 3 \ln z$

b) $2 \log x - 3 \log y + 2 \log z$

Example 3: (to be solved and finished in class) Solve for x:

a) $5^x = 10$

b) $\ln x = 4$

c) $3^x = 5$

d) $\log_3(2x-1) - \log_3(x-4) = 2$

e) $\log_3(x-4) + \log_3(x+4) = 3$

f) $\log x + \log(x-3) = 1$

g) $\log_2x + \log_2(x-2) = 3$

Example 4: (to be solved in class, but finished at home) Solve for x:

a) $\ln x = -2$

b) $\log_2x + \log_2(x-2) = 3$

c) $e^{t(t+1)} = 1$

d) $\log_4(x+6) - \log_4x = 2$

e) $\ln(2t+1) + \ln(2t-1) = 0$

f) $\ln(t-1) = 3$

g) $5e^{x-3} = 4$

h) $4^x = 5(2^x)$

i) $4(e^{2x}) = e^{6x}$

Answers (not on order): $(2/5)$; (4) ; $(1/e^2)$; (e^3+1) ; $(0, -1)$; $(\frac{1}{\sqrt{2}})$; $(3+\ln 0.8)$; (0.3466) ; (2.322)

Homework To Be Turned In:

Solve for t using natural logarithms:

1) $e^t = 100$

2) $e^t = 60$

3) $e^{-0.02t} = 0.06$

4) $\ln t = 2$

5) $\ln t = -3$

6) $e^{0.07t} = 2$

7) $2^t = 43$

8) $4^t = 8$

9) $6^t = 10$

10) $(5.2)^t = 70$

11) $2e^{3t} = 8e^{2t}$

12) $4 = 4e^{0.5t}$

13) $\ln(3t+1) - \ln(2t+3) = 0$

14) $4(3^t) = 5(2^t)$

15) $5e^{3t} = 8e^{2t}$

16) $\ln(3t+1) + \ln(3t-1) = 0$

17) $\ln(t-1) = 2$

18) $6(5^t) = 8(2^t)$

Answers (not on order):

(141) , (9.9021) , (1.2851) , (1.3863) , (0.47) , (4.6) , (2.5769) , (4.1) , (1.5) , (5.4263) , (0) ,
 (e^2) , (e^{-3}) , (0.55) , (0.314) , $(\sqrt{2}/3)$, (8.39) , (2)