MATH 119 **Integrals and Integral Applications (Class Note)**

Section 7.1: Integral Formulas (Antiderivative):

In Derivative, you multiply by the original power and you subtract 1 from the original power.

Example: $y = x^4 + 2x^3 + 5$ Then: $y' = 4x^3 + 6x^2 + 0$

In Antiderivative we do the opposite: Add 1 to the power, divide by the new power

If $y' = 4x^3 + 6x^2$; then $y = \int (4x^3 + 6x^2)dx = \frac{4x^4}{4} + \frac{6x^3}{3} = x^4 + 2x^3 + c$

the c is added for the 5 in the original problem

Derivative (Sections 4.1, 4.2)	**Integral (Section 7.1)**
• If $y = x^a$; then $y' = ax^{a-1}$	• $\int x^a dx = \frac{x^{a+1}}{a+1} + c$
• If $y = x$; then $y' = 1$ *Exmaple*: $y = 400\,x$, $y' = 400$	• $\int dx = x + c$ $y' = 400x$, then $y = \int 400 dx = 400 \int dx = 400x + c$
• If $y = \ln x$; then $y' = \frac{1}{x}$ *Exmaple*: $y = 100 \ln x$, $y' = \frac{100}{x}$	• $\int \frac{dx}{x} = \ln x + c$ $y' = \frac{100}{x}$, then $y = \int \frac{100}{x} dx = 100 \int \frac{dx}{x} = 100 \ln x + c$
• If $y = e^{ax}$; then $y' = ae^{ax}$ *Exmaple*: $y = \frac{1}{5} e^{5x}$, $y' = e^{5x}$	• $\int e^{ax} = \frac{1}{a} e^{ax} + c$ $y' = e^{5x}$, then $y = \int e^{5x} dx = \frac{1}{5} e^{5x} + c$
• If $y = a^x$; then $y' = \ln a . a^x$ *Exmaple*: $y = \frac{1}{\ln 2} .2^x$, $y' = \frac{1}{\ln 2} . \ln 2 . 2^x = 2^x$	• $\int a^x = \frac{a^x}{\ln a} + c$ $y' = 2^x$, then $y = \int 2^x dx = \frac{1}{\ln 2} .2^x + c$

Section 7.3: Definite Integrals

$$\int_a^b F'(x)dx = F(b) - F(a)$$

Example: $\int_1^2 \frac{4}{x} dx = 4[\ln x]_1^2 = 4[\ln 2 - \ln 1] = 4 . \ln 2 = 2.773$

Remember: $\ln 1 = 0$; $\ln e = 1$; $e^0 = 1$

Section 6.3: Annuities, Future and Present Value of an Income Stream:

In chapter 1, we covered the present and future value of a single payment. Now we see how to calculate the present and future value of a continuous stream of payment as an income or investment *(annuity)*.

Here are some examples for each:

- **Annuity (*A*)**: an investment each month or year in the bank for a future college fund, IRA…
- **Continuous income (*A*):** an income generated each month or year in a business such as monthly rent, house payments, memberships dues…
- **Present Value (P_0)** : The amount of money that must be deposited today to generate the same income stream over the same term . Lottery: cash option now, or payments.

	Single Payment (Sec. 1.6, 1.7)	***Continuous Stream of payment or Annuity (A) (Sec. 6.3)***
Future Value	$P = P_0 e^{rt}$	$F = \int_0^t A.e^{rt} dt = A\int_0^t e^{rt} dt$
Present Value	$P_0 = Pe^{-rt}$	$P = \int_0^t A.e^{-rt} dt = A\int_0^t e^{-rt} dt$

Reminder: $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$ **;** ***example:*** $\int_0^2 e^{-4x} dx = \frac{1}{-4}\left[e^{-4x}\right]_0^2 = \frac{-1}{4}\left[e^{-8} - e^0\right] = 0.2499$

Ex. 1: Find the present and future value of a constant income stream of \$1000 per year over a period of 20 years, assuming an interest rate of 6% compounded continuously.

(Ans: Present value = \$11,646.76 ; Future value = \$38,668.62)

Ex. 2: Suppose you want to have \$50,000 in 8 years in a bank account with 2% interest rate compounded continuously.

a) If you make one lump sum deposit now, how much should you deposit?

b) If you deposit money continuously throughout the 8-year period, how much should you deposit each year, each month?

(Ans: a) \$42,607.20 ; b) A = \$5763.33 per year or \$480 per month)

Ex. 3: If an amount of \$1000 was invested in the bank every year for 10 years with 8% interest compounded continuously. Find the new balance *(value of annuity)* after 10 years.

(Ans: \$ 15319.27)

Ex. 4: What should *A (annuity)* per year be so that the amount of a continuous money flow over 25 years at interest rate 12%, compounded continuously, will be \$ 40,000?

(Ans: A = \$251.50)

Ex. 5: A new department store is expected to generate income at continuous rate of \$50,000 per year over the next 5 years. Find the present value of the store if the current interest rate is 10% compounded continuously

(Ans: \$196734.67).

Ex. 6: Find the accumulated present value of an investment over 20 years period if there is a continuous money flow of \$1800 per year and the current interest rate is 8% compounded continuously.

(Ans: \$17,957.32)