

Example 8: A marketing analysis shows that 63% of the consumers who currently drink Coke will purchase Coke the next time, and 12% of consumers who drink Pepsi will switch to Coke. Find the steady state vector.

$$T = \begin{matrix} & \begin{matrix} C & P \end{matrix} \\ \begin{matrix} C \\ P \end{matrix} & \begin{bmatrix} 0.63 & 0.37 \\ 0.12 & 0.88 \end{bmatrix} \end{matrix} = \begin{matrix} 1 \\ 1 \end{matrix}$$

$$P = [x \quad y]$$

$$[x \quad y] \cdot \begin{bmatrix} 0.63 & 0.37 \\ 0.12 & 0.88 \end{bmatrix} = [x \quad y]$$

$$\begin{aligned} 0.63x + 0.12y &= x \longrightarrow 0.63x - x + 0.12y = 0 \\ 0.37x + 0.88y &= y \longrightarrow 0.37x + 0.88y - y = 0 \end{aligned}$$

$$-0.37x + 0.12y = 0 \longrightarrow -37x + 12y = 0$$

$$0.37x - 0.12y = 0 \longrightarrow 37x - 12y = 0$$

$$x + y = 1 \longrightarrow x + y = 1$$

X	Y	
-37*	12	0
1	1	1
-37	12	0
0	-49*	-37
-49	0	-12
0	-49	-37
1	0	0.24
0	1	0.76

$$\frac{(0) - (12)(-37)}{-37} = -12$$

$$\text{Sum} = 1$$

$$X = 24\%$$

$$Y = 76\%$$

Coke

Pepsi

Example 9: An extensive survey of customers of three major cable companies (A, B and C) found the following:

Company A will keep 71% of its customers, 12% will move to B and the rest will move to C.

Company B will lose 32% of its customer to A and 34% to C.

Company C will keep 96% of its customers with half of the rest moving to A and half to B.

Find the steady state vector

$$T = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0.71 & 0.12 & 0.17 \\ 0.32 & 0.34 & 0.34 \\ 0.02 & 0.02 & 0.96 \end{bmatrix} \end{matrix} = \begin{matrix} 1 \\ 1 \\ 1 \end{matrix}$$

$$P = [x \quad y \quad z]$$

$$[x \quad y \quad z] \cdot \begin{bmatrix} 0.71 & 0.12 & 0.17 \\ 0.32 & 0.34 & 0.34 \\ 0.02 & 0.02 & 0.96 \end{bmatrix} = [x \quad y \quad z]$$

$$\begin{array}{l} 0.71x + 0.32y + 0.02z = x \rightarrow -0.29x + 0.32y + 0.02z = 0 \\ 0.12x + 0.34y + 0.02z = y \rightarrow 0.12x - 0.66y + 0.02z = 0 \\ 0.17x + 0.34y + 0.96z = z \rightarrow 0.17x + 0.34y - 0.04z = 0 \\ \hline = 1 \qquad \qquad = 1 \qquad \qquad = 1 \\ x + y + z = 1 \end{array}$$

$$12x - 66y + 2z = 0$$

$$17x + 34y - 4z = 0$$

$$x + y + z = 1$$

x	y	z	
12*	-66	2	0
17	34	-4	0
1	1	1	1
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12	-66	2	0
0	1530*	-82	0
0	78	10	12
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1530	0	-196	0
0	1530	-82	0
0	0	1808*	1530
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1808	0	0	196
0	1808	0	82
0	0	1808	1530

$$\frac{(0) - (-196)(1530)}{1530} = 196$$

x	y	z	
1	0	0	0.1084
0	1	0	0.0454
0	0	1	0.8462

} Sum = 1

$X = 10.84\%$ A	$Y = 4.54\%$ B	$Z = 84.62\%$ C
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