Chapter 6: Linear Equations and Matrix Algebra

Section 6.3 Cont.: Inverse Matrix

To find the inverse matrix of $A = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$ using the All Integer Method:

- Step 1: Re-write it with the Identity Matrix I next to it on the right side:
  (The Identity Matrix I: the square matrix where all Diagonal elements = 1, the rest are zeros)

$$\left|\begin{array}{cc:cc} 2^* & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right|$$

$$\quad A \qquad\quad I$$

- Step 2: Do the pivot steps (2 pivots for two rows), and the last step should be:

$$\left|\begin{array}{cc:cc} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \end{array}\right|$$

$$\quad I \qquad\quad A^{-1}$$

- Step 3: The Identity Matrix I is now on the left side, and the Inverse Matrix A^{-1} is on the right side:

$$A^{-1} = \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix}$$

$$A \cdot A^{-1} = \underline{I}$$

- You can check your answer by multiplying the original matrix A and the inverse A^{-1}. The answer must be an Identity Matrix I.

Note: Not every matrix has an inverse, for example: $A = \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix}$ does not have an inverse (the second pivot is zero).

Solve the same example and show steps

$A = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$ find A^{-1}

$$\begin{array}{cc|cc} 2^* & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ \hline \multicolumn{2}{c|}{A} & \multicolumn{2}{c}{I} \end{array}$$

$$\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 0 & 1^* & -1 & 2 \\ \hline 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ \hline \multicolumn{2}{c|}{I} & \multicolumn{2}{c}{A^{-1}} \end{array}$$

$$\frac{(1) - (-1)}{2} = 1$$

$$A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Examples: Find the inverse matrix for each of the following and check your answer by multiplying the original matrix by its inverse, the resulting matrix must be an Identity Matrix:

1) $\begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix}$

2) $\begin{vmatrix} 1 & 0 \\ 3 & -2 \end{vmatrix}$

1)

A		I	
1*	1	1	0
2	4	0	1
1	1	1	0
0	2*	-2	1
2	0	4	-1
0	2	-2	1
1	0	2	$-\frac{1}{2}$
0	1	-1	$\frac{1}{2}$
I		A^{-1}	

2)

1*	0	1	0
3	-2	0	1
1	0	1	0
0	-2*	-3	1
-2	0	-2	0
0	-2	-3	1
1	0	1	0
0	1	$\frac{3}{2}$	$-\frac{1}{2}$
I		A^{-1}	

3) $\begin{vmatrix} 4 & -1 \\ 3 & -1 \end{vmatrix}$

$$\left[\begin{array}{cc|cc} 4^* & -1 & 1 & 0 \\ 3 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 4 & -1 & 1 & 0 \\ 0 & -1^* & -3 & 4 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} -1 & 0 & -1 & 1 \\ 0 & -1 & -3 & 4 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & 3 & -4 \end{array}\right]$$

$I \mid A^{-1}$

4) $\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$

$$\left[\begin{array}{cc|cc} 2^* & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 0 & 1^* & -3 & 2 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & 1 & -3 & 2 \end{array}\right]$$

$I \mid A^{-1}$

<u>Note</u>: Not every matrix has an inverse, for example: $A = \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix}$ does not have an inverse (the second pivot is zero).

$$\left[\begin{array}{cc|cc} 2^* & 1 & 1 & 0 \\ 4 & 2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 0 & 0 & -4 & 2 \end{array}\right]$$

$$\begin{array}{c|c} A & \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \\ \hline \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} & A^{-1} \end{array}$$

Find the inverse of the following 3x3 matrix:

$$A = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

Original Matrix A			*Identity matrix I*		
2	1	1	1	0	0
1	2	-1	0	1	0
1	1	1	0	0	1
2	1	1	1	0	0
0	3	-3	-1	2	0
0	1	1	-1	0	2
3	0	3	2	-1	0
0	3	-3	-1	2	0
0	0	3	-1	-1	3
3	0	0	3	0	-3
0	3	0	-2	1	3
0	0	3	-1	-1	3
1	0	0	1	0	-1
0	1	0	- 2/3	1/3	1
0	0	1	- 1/3	- 1/3	1
	I			A^{-1}	