Chapter 6: Linear Equations and Matrix Algebra

<u>Section 6.3:</u> **Matrix Notation**

Matrices

- A *matrix* is a rectangular or square array of values arranged in rows and columns.
- An $m \times n$ matrix $\mathbf{A}$, has m rows and n columns, and has a general form of

$$\mathbf{A}_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

Row by Column

$m \times n$

Examples of Matrices

$$\mathbf{A} = \begin{bmatrix} 3 & 5 & 8 \\ 1 & 2 & 2 \end{bmatrix}$$

2×3

$a_{21} = 1$

$a_{12} = 5$

$$\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 3 & 6 \\ 7 & 2 \\ 2 & 9 \end{bmatrix}$$

4×2

$b_{21} = 3$

$b_{12} = 0$

$$C = \begin{vmatrix} -1 \\ 2 \\ 1 \\ 3 \end{vmatrix}$$

4×1

$c_{41} = 3$

$c_{14} = N.P$

$$D = \begin{vmatrix} -1 & 2 & 1 & 4 \end{vmatrix}$$

1×4

$d_{41} = N.P.$

$d_{14} = 4$

Two matrices are equal if they have same <u>dimension</u>, same <u>elements</u>:

$$A = \begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} \quad ; \quad B = \begin{vmatrix} 1 & x \\ -1 & y \end{vmatrix}$$

2×2 $\qquad$ 2×2

If $A = B$, then:

$2 = x$

$3 = y$

Addition of Matrices A and B:

A and B must have same dimensions:

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 8 \\ 9 & -1 & 5 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 5 & 3 \\ 2 & 3 & -7 \end{bmatrix}$$

2×3 $\qquad$ 2×3

$$\mathbf{A}+\mathbf{B} = \begin{bmatrix} 2+1 & 4+5 & 8+3 \\ 9+2 & (-1)+3 & 5+(-7) \end{bmatrix} = \begin{bmatrix} 3 & 9 & 11 \\ 11 & 2 & -2 \end{bmatrix}$$

$$\mathbf{A}-\mathbf{B} = \begin{bmatrix} 2-1 & 4-5 & 8-3 \\ 9-2 & -1-3 & 5-(-7) \end{bmatrix} = \begin{bmatrix} 1 & -1 & 5 \\ 7 & -4 & 12 \end{bmatrix}$$

If k is a real number, then the scalar product $k.A$ is abstained by multiplying each element of A by k

If k = -2 and $A = \begin{vmatrix} -1 & -4 & 5 \\ 1 & 3 & -3 \\ 2 & 4 & -2 \end{vmatrix}$

Then $k.A$ = -2.A = $\begin{pmatrix} 2 & 8 & -10 \\ -2 & -6 & 6 \\ -4 & -8 & 4 \end{pmatrix}$

Example: for the following matrices, find $2A - 3B$

$$A = \begin{vmatrix} -1 & -4 & 5 \\ 1 & 3 & -3 \\ 2 & 4 & -2 \end{vmatrix} \qquad B = \begin{vmatrix} 4 & -1 & 2 \\ -1 & -2 & 3 \\ -3 & -4 & 1 \end{vmatrix}$$

$$2A - 3B = \underbrace{\begin{pmatrix} -2 & -8 & 10 \\ 2 & 6 & -6 \\ 4 & 8 & -4 \end{pmatrix}}_{2A} \; \underbrace{-}_{-} \; \underbrace{\begin{pmatrix} 12 & -3 & 6 \\ -3 & -6 & 9 \\ -9 & -12 & 3 \end{pmatrix}}_{3B}$$

$$= \boxed{\begin{pmatrix} -14 & 5 & 4 \\ 5 & 12 & -15 \\ 13 & 20 & -7 \end{pmatrix}}$$

The Transpose of a Matrix:

Each row becomes column, and each column becomes row.

$$A = \begin{vmatrix} -1 & 4 \\ 1 & 3 \\ 2 & 4 \end{vmatrix}$$

3×2

$$A^t = \begin{vmatrix} -1 & 1 & 2 \\ -4 & 3 & 4 \end{vmatrix}$$

2×3