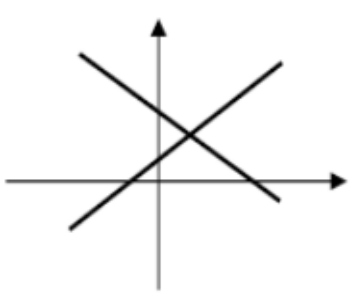
Chapter 6: Linear Equations and Matrix Algebra

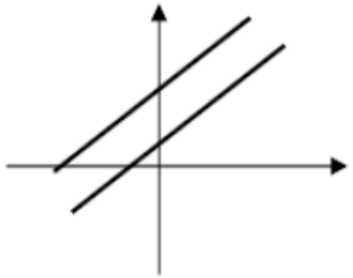
Section 6.1 and 6.2: The All Integer Method

- **Method 1:** Solving Graphically



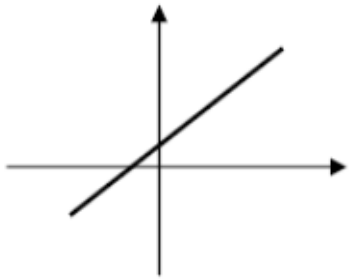
Consistent, Independent

One solution



Inconsistent, Independent

No solution



Consistent, Dependent

(y depends on x)

Infinite number of solutions

- **Method 2:** Solving by **Substitution**
- **Method 3:** Solving by **Elimination**
- **Method 4:** Solving by **The All Integer Echelon Method**

The All Integer Method First and Last Steps:

Example:

Solve for x, y and z:

$$2x + y + 3z = 5$$
$$2y - z = 5$$
$$2x + y + 2z = 6$$

*The First step is to create the Setup (initial) table:

x	y	z	
2	1	3	5
0	2	-1	5
2	1	2	6

*The Last step will be:

x	y	z	
1	0	0	3
0	1	0	2
0	0	1	-1

*The Answer : $x = 3$, $y = 2$ and $z = -1$

The All Integer Method Steps:

Solve for x, y and z using the All-Integers Method:

$2x + y + 3z = 5$

$2y - z = 5$

$2x + y + 2z = 6$

x	y	z	
2*	1	3	5
0	2	-1	5
2	1	2	6
2*	1	3	5
0			
0			

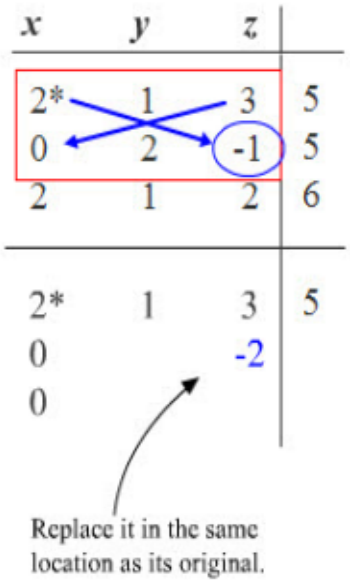
1) Setup the initial table and select the first Pivot Element (* *the first element in the table*).

2) Copy the pivot row and make all other elements in the pivot column = 0.

3) Replace the other elements using the "criss-cross" multiplication method.

"Criss-Cross" Operation Step by Step Using First Pivot

Current Pivot (*First Pivot*) = 2 ; **Previous** Pivot (*there is none, so assume it* = 1)



Lets replace any element such as -1:

Create a rectangle where the pivot element and the element to be replaced are on facing corners.

Multiply the pivot element by the element to be replaced (2*)(-1)

Subtract the product of the two elements on the opposite diagonal. - (3)(0)

Divide the result by the previous pivot element

$$\frac{(2^*)(-1) - (3)(0)}{\textit{previous pivot}} = \frac{-2}{1} = -2$$

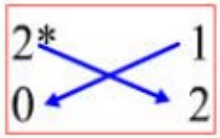
Note: The result must be an **Integer** (no decimal, no fraction) until the very last step (as we will see later).

First Tableau, First Pivot

Current Pivot (*First Pivot*) = 2

Previous Pivot (*there is none, so assume it* = 1)

x	*y*	*z*	
2*	1	3	5
0	2	-1	5
2	1	2	6



2*	1	3	5
0	4	-2	
0			

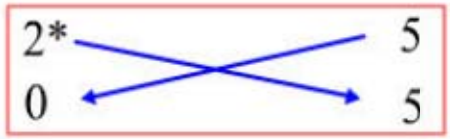
$$\frac{(2^*)(2) - (1)(0)}{\textit{previous pivot}} = \frac{4}{1} = 4$$

First Tableau, First Pivot

Current Pivot (*First Pivot*) = 2

Previous Pivot (*there is none, so assume it* = 1)

x	*y*	*z*	
2*	1	3	5
0	2	-1	5
2	1	2	6
2*	1	3	5
0	4	-2	10
0			



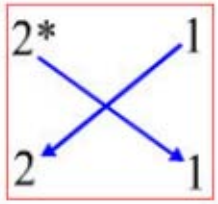
$$\frac{(2^*)(5) - (5)(0)}{\textit{previous pivot}} = \frac{10}{1} = 10$$

First Tableau, First Pivot

Current Pivot (*First Pivot*) = 2

Previous Pivot (*there is none, so assume it* = 1)

x	*y*	*z*	
2*	1	3	5
0	2	-1	5
2	1	2	6



2*	1	3	5
0	4	-2	10
0	0		

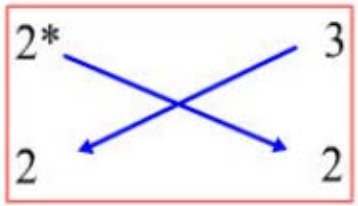
$$\frac{(2^*)(1) - (1)(2)}{\textit{previous pivot}} = \frac{0}{1} = 0$$

First Tableau, First Pivot

Current Pivot (*First Pivot*) = 2

Previous Pivot (*there is none, so assume it* = 1)

x	*y*	*z*	
2*	1	3	5
0	2	-1	5
2	1	2	6



2*	1	3	5
0	4	-2	10
0	0	-2	

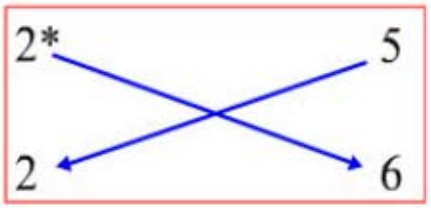
$$\frac{(2^*)(2) - (3)(2)}{\text{previous pivot}} = \frac{-2}{1} = -2$$

First Tableau, First Pivot

Current Pivot (*First Pivot*) = 2

Previous Pivot (*there is none, so assume it* = 1)

x	*y*	*z*	
2*	1	3	5
0	2	-1	5
2	1	2	6



2*	1	3	5
0	4	-2	10
0	0	-2	2

$$\frac{(2^*)(6) - (5)(2)}{\text{previous pivot}} = \frac{2}{1} = 2$$

First Tableau, First Pivot

Current Pivot (*First Pivot*) = 2
Previous Pivot (*there is none, so assume it* = 1)

x	*y*	*z*	
2*	1	3	5
0	2	-1	5
2	1	2	6
2*	1	3	5
0	4	-2	10
0	0	-2	2

First Tableau Finished

Second Tableau, Second Pivot

Select a new pivot element which is located diagonaly in the next row.

x	*y*	*z*	
2	1	3	5
0	4*	-2	10
0	0	-2	2
4	0	.	.
0	4*	-2	10
0	0	.	.

New (Second) Pivot = 4

Previous Pivot = 2

Repeat the steps used in the previous table:

Copy the pivot row and make all other elements in the pivot column = 0.

Note: For columns already pivoted, the old pivot will change to the new and current pivot and the 0's will stay.

Replace the other elements using the "criss-cross" multiplication method.

$$\frac{(4)(2) - (1)(0)}{2}$$

$$= \frac{8}{2} = 4$$

Second Tableau, Second Pivot

New (Second) Pivot = 4

Previous Pivot = 2

x	*y*	*z*	
2	1	3	5
0	4*	-2	10
0	0	-2	2
4	0	7	5
0	4*	-2	10
0	0	-4	4

Second Tableau Finished

$$\frac{(4)(3)-(-2)(1)}{2} = 7$$

$$\frac{(4)(5)-(10)(1)}{2} = 5$$

$$\frac{(4)(-2)-(0)(-2)}{2}$$

$$= -4$$

$$\frac{(4)(2)-(10)(0)}{2} = 4$$

Third Tableau, Third Pivot

Select a new pivot element which is located diagonaly in the next row.

x	*y*	*z*	
4	0	7	5
0	4	-2	10
0	0	-4*	4
-4	0	0	-12
0	-4	0	-8
0	0	-4*	4

New (Third) Pivot = -4
Previous Pivot = 4

Repeat the steps used in the previous table:

Copy the pivot row and make all other elements in the pivot column = 0.

Note: For columns already pivoted, the old pivot will change to the new and current pivot and the 0's will stay.

Replace the other elements using the "criss-cross" multiplication method.

$$\frac{-16-(0)}{4} = -4$$

$$\frac{(-4)(5)-(7)(7)}{4}$$

$$\frac{-20-14}{4}$$

$$= -12$$

$$\frac{(-4)(10)-(4)(-2)}{4}$$

$$= -8$$

Third Tableau, Third Pivot

New (Third) Pivot = -4
Previous Pivot = 4

x	*y*	*z*	
4	0	7	5
0	4	-2	10
0	0	-4*	4
-4	0	0	-12
0	-4	0	-8
0	0	-4*	4

Third Tableau Finished

Last Tableau

Select a new pivot element which is located diagonaly in the next row.

x	y	z	
-4	0	0	-12
0	-4	0	-8
0	0	-4	4
1	0	0	3
0	1	0	2
0	0	1	-1

New Pivot = No more rows

No more pivot

Previous or last Pivot = -4

Divide all elements by the last pivot wich is = -4.

*This is the last step and the **only step** where you can get fractions or decimals as answers.*

$$x = 3$$

$$y = 2$$

$$z = -1$$

Summary of All Tableaus and Pivots

x	y	z		
2	1	3	5	
0	2	-1	5	Setup
2	1	2	6	
2*	1	3	5	
0	4	-2	10	First
0	0	-2	2	
4	0	7	5	
0	4*	-2	10	Second
0	0	-4	4	
-4	0	0	-12	
0	-4	0	-8	Third
0	0	-4*	4	

Last Tableau, Result:

x	y	z	
1	0	0	3
0	1	0	2
0	0	2	-1

$$x = 3$$

$$y = 2$$

$$z = -1$$