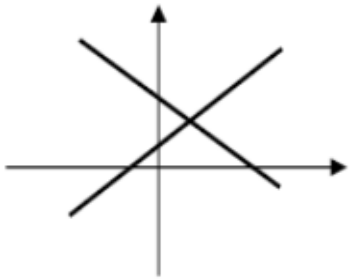
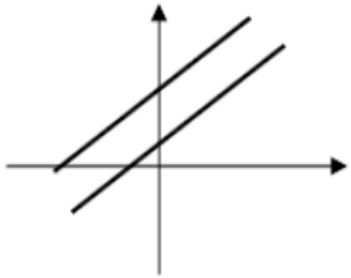
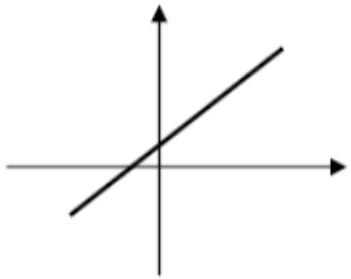
Chapter 6: Linear Equations and Matrix Algebra

Section 6.1 and 6.2: The All Integer Method

- **Method 1:** Solving Graphically

		
Consistent, Independent One solution	Inconsistent, Independent No solution	Consistent, Dependent (y depends on x) Infinite number of solutions

- **Method 2:** Solving by **Substitution**
- **Method 3:** Solving by **Elimination**
- **Method 4:** Solving by **The All Integer Echelon Method**

The All Integer Method First and Last Steps:

Example:

Solve for x, y and z:

$$2x + y + 3z = 5$$
$$2y - z = 5$$
$$2x + y + 2z = 6$$

*The First step is to create the Setup (initial) table:

x	y	z	
2	1	3	5
0	2	-1	5
2	1	2	6

*The Last step will be:

x	y	z	
1	0	0	3
0	1	0	2
0	0	1	-1

*The Answer : $x = 3$, $y = 2$ and $z = -1$

The All Integer Method Steps:

Solve for x, y and z using the All-Integers Method:

$$2x + y + 3z = 5$$
$$2y - z = 5$$
$$2x + y + 2z = 6$$

x	y	z	
2*	1	3	5
0	2	-1	5
2	1	2	6

1) Setup the initial table and select the first Pivot Element (* *the first element in the table*).

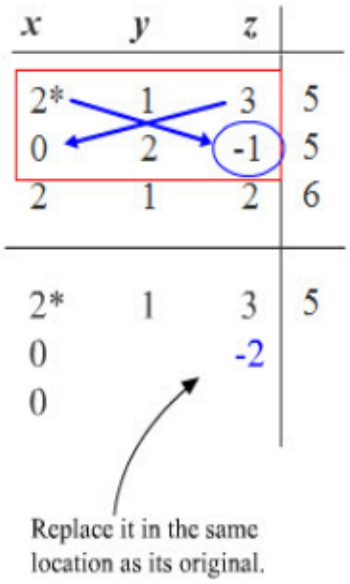
2*	1	3	5
0			
0			

2) Copy the pivot row and make all other elements in the pivot column = 0.

3) Replace the other elements using the "criss-cross" multiplication method.

"Criss-Cross" Operation Step by Step Using First Pivot

Current Pivot (*First Pivot*) = 2 ; **Previous** Pivot (*there is none, so assume it* = 1)



Lets replace any element such as -1:

Create a rectangle where the pivot element and the element to be replaced are on facing corners.

Multiply the pivot element by the element to be replaced (2*)(-1)

Subtract the product of the two elements on the opposite diagonal. - (3)(0)

Divide the result by the previous pivot element

$$\frac{(2^*)(-1) - (3)(0)}{\textit{previous pivot}} = \frac{-2}{1} = -2$$

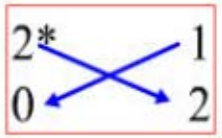
Note: The result must be an **Integer** (no decimal, no fraction) until the very last step (as we will see later).

First Tableau, First Pivot

Current Pivot (*First Pivot*) = 2

Previous Pivot (*there is none, so assume it* = 1)

x	*y*	*z*	
2*	1	3	5
0	2	-1	5
2	1	2	6



2*	1	3	5
0	4	-2	
0			

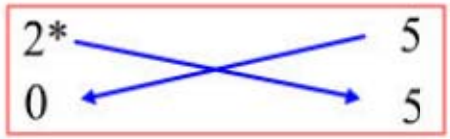
$$\frac{(2^*)(2) - (1)(0)}{\textit{previous pivot}} = \frac{4}{1} = 4$$

First Tableau, First Pivot

Current Pivot (*First Pivot*) = 2

Previous Pivot (*there is none, so assume it* = 1)

x	*y*	*z*	
2*	1	3	5
0	2	-1	5
2	1	2	6
2*	1	3	5
0	4	-2	10
0			



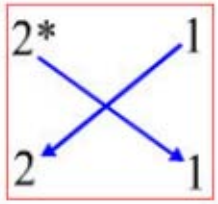
$$\frac{(2^*)(5) - (5)(0)}{\textit{previous pivot}} = \frac{10}{1} = 10$$

First Tableau, First Pivot

Current Pivot (*First Pivot*) = 2

Previous Pivot (*there is none, so assume it* = 1)

x	*y*	*z*	
2*	1	3	5
0	2	-1	5
2	1	2	6



2*	1	3	5
0	4	-2	10
0	0		

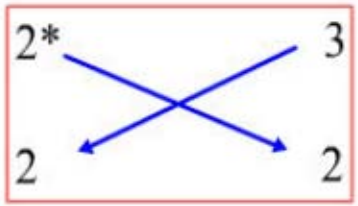
$$\frac{(2^*)(1) - (1)(2)}{\text{previous pivot}} = \frac{0}{1} = 0$$

First Tableau, First Pivot

Current Pivot (*First Pivot*) = 2

Previous Pivot (*there is none, so assume it* = 1)

x	*y*	*z*	
2*	1	3	5
0	2	-1	5
2	1	2	6



2*	1	3	5
0	4	-2	10
0	0	-2	

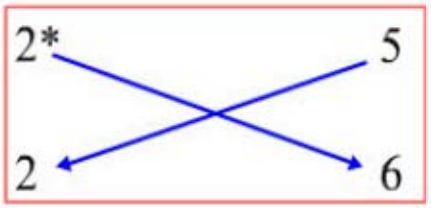
$$\frac{(2^*)(2) - (3)(2)}{\text{previous pivot}} = \frac{-2}{1} = -2$$

First Tableau, First Pivot

Current Pivot (*First Pivot*) = 2

Previous Pivot (*there is none, so assume it* = 1)

x	*y*	*z*	
2*	1	3	5
0	2	-1	5
2	1	2	6



2*	1	3	5
0	4	-2	10
0	0	-2	2

$$\frac{(2^*)(6) - (5)(2)}{\text{previous pivot}} = \frac{2}{1} = 2$$

First Tableau, First Pivot

Current Pivot (*First Pivot*) = 2
Previous Pivot (*there is none, so assume it* = 1)

x	*y*	*z*	
2*	1	3	5
0	2	-1	5
2	1	2	6
2*	1	3	5
0	4	-2	10
0	0	-2	2

First Tableau Finished

Second Tableau, Second Pivot

Select a new pivot element which is located diagonaly in the next row.

x	y	z	
2	1	3	5
0	4*	-2	10
0	0	-2	2
4	0		
0	4*	-2	10
0	0		

New (Second) Pivot = 4
Previous Pivot = 2

Repeat the steps used in the previous table:

Copy the pivot row and make all other elements in the pivot column = 0.

Note: For columns already pivoted, the old pivot will change to the new and current pivot and the 0's will stay.

Replace the other elements using the "criss-cross" multiplication method.

Second Tableau, Second Pivot

New (Second) Pivot = 4
Previous Pivot = 2

x	*y*	*z*	
2	1	3	5
0	4*	-2	10
0	0	-2	2
4	0	7	5
0	4*	-2	10
0	0	-4	4

Second Tableau Finished

Third Tableau, Third Pivot

Select a new pivot element which is located diagonaly in the next row.

x	*y*	*z*	
4	0	7	5
0	4	-2	10
0	0	-4*	4
-4	0	0	
0	-4	0	
0	0	-4*	4

New (Third) Pivot = -4
Previous Pivot = 4

Repeat the steps used in the previous table:

Copy the pivot row and make all other elements in the pivot column = 0.

Note: For columns already pivoted, the old pivot will change to the new and current pivot and the 0's will stay.

Replace the other elements using the "criss-cross" multiplication method.

Third Tableau, Third Pivot

New (Third) Pivot = -4
Previous Pivot = 4

x	*y*	*z*	
4	0	7	5
0	4	-2	10
0	0	-4*	4
-4	0	0	-12
0	-4	0	-8
0	0	-4*	4

Third Tableau Finished

Last Tableau

Select a new pivot element which is located diagonaly in the next row.

x	y	z	
-4	0	0	-12
0	-4	0	-8
0	0	-4	4
1	0	0	3
0	1	0	2
0	0	1	-1

New Pivot = No more rows
No more pivot
Previous or last Pivot = -4

Divide all elements by the last pivot wich is = -4.

*This is the last step and the **only step** where you can get fractions or decimals as answers.*

$$x = 3$$
$$y = 2$$
$$z = -1$$

Summary of All Tableaus and Pivots

x	y	z		
2	1	3	5	
0	2	-1	5	Setup
2	1	2	6	
2*	1	3	5	
0	4	-2	10	First
0	0	-2	2	
4	0	7	5	
0	4*	-2	10	Second
0	0	-4	4	
-4	0	0	-12	
0	-4	0	-8	Third
0	0	-4*	4	

Last Tableau, Result:

x	y	z	
1	0	0	3
0	1	0	2
0	0	2	-1

$$x = 3$$

$$y = 2$$

$$z = -1$$

The All Integer Method
Example 2

Solve for x, y and z using the all integers method:

$$3x + 2z = 9$$
$$x - y - 3z = -3$$
$$-x + 2y + 4z = 5$$

x	y	z	
3	0	2	9
1	-1	-3	-3
-1	2	4	5

Setup Table

Solve for x, y and z using the all integers method:

$$3x + 2z = 9$$
$$x - y - 3z = -3$$
$$-x + 2y + 4z = 5$$

x	y	z	
3	0	2	9
1	-1	-3	-3
-1	2	4	5

Setup Table

3*	0	2	9
0	-3	-11	-18
0	6	14	24

First Tableau
First Pivot = 3

-3	0	-2	-9
0	-3*	-11	-18
0	0	8	12

Second Tableau
Second Pivot = -3

8	0	0	16
0	8	0	4
0	0	8*	12

Third Tableau
Third Pivot = 8

x	y	z	
1	0	0	2
0	1	0	1/2
0	0	1	3/2

$$x = 2$$
$$y = 1/2$$
$$z = 3/2$$

The All Integer Method
Example 3

Solve for x and y using the all integers method:

$$2x - y = 4$$
$$x + y = 5$$

Solve for x and y using the all integers method:

$$2x - y = 4$$
$$x + y = 5$$

x	y		
2	-1	4	Setup Table
1	1	5	
2*	-1	4	First Tableau
0	3	6	First Pivot = 2
3	0	9	Second Tableau
0	3*	6	Second Pivot = 3
1	0	3	
0	1	2	

$x = 3$

$y = 2$

The All Integer Method , Special Cases:

Case 1, No Solution

Solve for x, y and z using the all integers method:

$$\begin{aligned} 2x - 6y + 4z &= 1 \\ 4x - 10y + 10z &= 3 \\ x - 2y + 3z &= 2 \end{aligned}$$

Solve for x, y and z using the all integers method:

$$\begin{aligned} 2x - 6y + 4z &= 1 \\ 4x - 10y + 10z &= 3 \\ x - 2y + 3z &= 2 \end{aligned}$$

x	y	z	
2	-6	4	1
4	-10	10	3
1	-2	3	2

Setup Table

2*	-6	4	1
0	4	4	2
0	2	2	3

First Tableau
First Pivot = 2

2	-6	4	1
0	4*	4	2
0	2	2	3

Second Tableau
Second Pivot = 4

4	0	20	8
0	4	4	2
0	0	0	4

No Solution.
Any time we have a row with all 0's to the left, and a nonzero to the right, the system is inconsistent or no solution. $0 = 4$ is not possible.

But, it is ok to have a zero on the right and nonzeros on the left (when variables are = 0). For example:

0	3	0	0

or: $3y = 0$

The All Integer Method , Special Cases:
Case 2, Infinite Number of Solutions

Solve for x, y and z using the all integers method:

$$\begin{aligned} 2x - 3y - 9z &= -5 \\ x + 3z &= 2 \\ -3x + y - 4z &= -3 \end{aligned}$$

Solve for x, y and z using the all integers method:

$$\begin{aligned} 2x - 3y - 9z &= -5 \\ x + 3z &= 2 \\ -3x + y - 4z &= -3 \end{aligned}$$

x	y	z	
2	-3	-9	-5
1	0	3	2
-3	1	-4	-3
2*	-3	-9	-5
0	3	15	9
0	-7	-35	-21
3	0	9	6
0	3*	15	9
0	0	0	0
1	0	3	2
0	1	5	3

Setup Table

First Tableau, First Pivot = 2

Second Tableau, Second Pivot = 3

Eliminate the line of all 0's.
Move to the next pivot if possible.

No more pivotes, then divide all elements by last pivot.
Translate the solution. It is **Linearly Dependent**

$$\begin{aligned} x + 3z &= 2 \\ y + 5z &= 3 \end{aligned}$$

or

$$\begin{aligned} x &= 2 - 3z \\ y &= 3 - 5z \\ z &= \text{any number} \end{aligned}$$