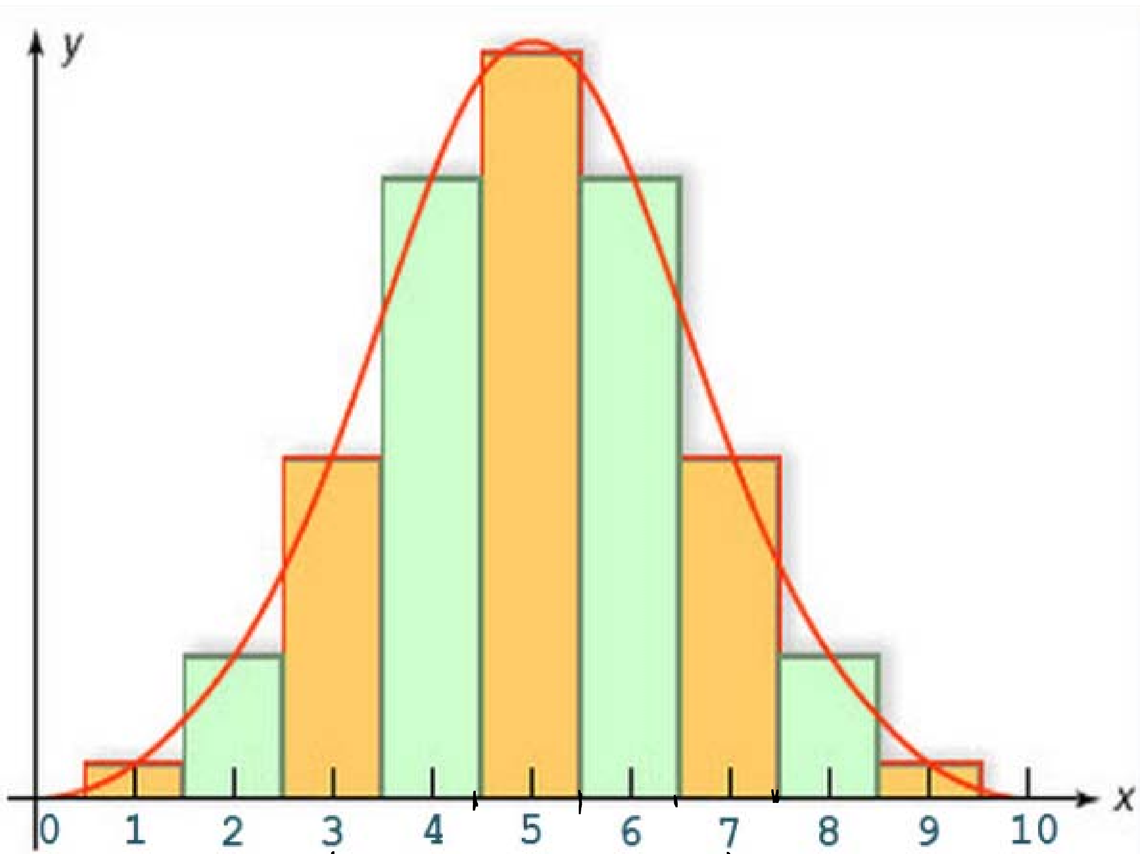
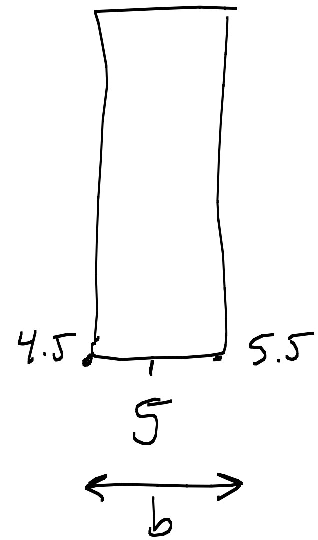
Section 5.4: **Normal Approximation To The Binomial**



$P(3 \le X \le 7)$

$P(2.5 \le X \le 7.5)$

$\cancel{P(3.5 \le X \le 7.5)}$



$P(X = 4)$

$P(3.5 \le X \le 4.5)$

$P(X \ge 6)$	$P(X \le 6)$
$P(X \ge 5.5)$	$P(X \le 6.5)$

± 0.5

<u>RULES:</u> To approximate **<u>binomial</u>** probability by normal curve area:

Step 1) determine n, p, q

Step 2) check that both $n.p > 5$ <u>and</u> $n.q > 5$

Step 3) find the expected value and the standard deviation

$$\mu = n \cdot p \qquad\qquad \sigma = \sqrt{n.p.q}$$

Step 4) find the new points by:

* subtracting 0.5 from the starting point
* adding 0.5 to the finish point

examples:

$P(3 \leq X \leq 6)$	*will be*	$P(2.5 \leq X \leq 6.5)$
$P(X = 7)$	*will be*	$P(6.5 \leq X \leq 7.5)$
$P(X \geq 8)$	*will be*	$P(X \geq 7.5)$
$P(X \leq 8)$	*will be*	$P(X \leq 8.5)$

Step 5) find the Z-scores and the area under the normal curve using the table

Example 1: According to the Department of Health and Human Services, the probability is about 80% that a person aged 70 will be alive at the age of 75. Suppose that 500 people aged 70 are selected at random. Find the probability that:

a) exactly 390 of them will be alive at the age of 75

a) Step 1) $n = 500$, $p = 0.8$, $q = 0.2$

Step 2) check if both $n.p$ and $n.q$ are more than 5:

$$\left.\begin{aligned} n.p &= (500).(0.8) = 400 \\ n.q &= (500).(0.2) = 100 \end{aligned}\right\} > 5$$

Step 3) find the expected value and the std. deviation:

$$\mu = n \cdot p = (500).(0.8) = 400$$

$$\sigma = \sqrt{n.p.q} = \sqrt{(500).(0.8).(0.2)} = 8.94$$

Step 4) find the new point:

$P(X = 390)$ will be $P(389.5 \le X \le 390.5)$

Step 5) find the Z-score:

$X = 389.5$, $\quad Z = \dfrac{389.5 - 400}{8.94} = -1.17$

$X = 390.5$, $\quad Z = \dfrac{390.5 - 400}{8.94} = -1.06$

and now by using the table:

$P(-1.17 \le Z \le -1.06) = 0.1446 - 0.1210 = 0.0236$

Example 1 (Cont.) : According to the Department of Health and Human Services, the probability is about 80% that a person aged 70 will be alive at the age of 75. Suppose that 500 people aged 70 are selected at random. Find the probability that:

$\mu = 400,$ $\sigma = 8.94$

b) for $P(375 \le X \le 425)$, we use the information of steps 1, 2 and 3 then:

$$P(375 \le X \le 425) \text{ will be } P(374.5 \le X \le 425.5)$$

$$X = 374.5, \qquad Z = \frac{374.5 - 400}{8.94} = -2.85$$

$$X = 425.5, \qquad Z = \frac{425.5 - 400}{8.94} = 2.85$$

and now by using the table:

$$P(-2.85 \le Z \le 2.85) = 0.9978 - .0022 = 0.9956$$

99.56%

	Section 5.3 No Approximation	**Section 5.4** Approximation
Given	Expected value μ Standard Deviation σ	n, p
Steps	• Find: Z-Score: $Z = \frac{X-\mu}{\sigma}$ • Use the table	• Find: q *where* $q = 1 - p$ excepted value $E[X] = \mu = n.p$ Standard deviation $\sigma = \sqrt{n.p.q}$ • Add / subtract 0.5 as needed • Find the Z-Score : $Z = \frac{X-\mu}{\sigma}$ • Use the table

$np > 5$

$nq > 5$

Example 2: A coin with Pr[Tails] = 0.4 is flipped 200 times. Find the probability of getting between 65 and 100 tails on the coin. Give your answer as a decimal number correct to three decimal places

given: $P(T) = 0.4$, $n = 200$, $q = 0.6$

$\boxed{\begin{array}{l} n \cdot p > 5 \\ n q > 5 \end{array}}$

Find: $P(65 \leq X \leq 100)$

$* \ \mu = 200(0.4) = 80$

$* \ \sigma = \sqrt{200(0.4)(0.6)} = \sqrt{48} = 6.9282$

$P(64.5 \leq X \leq 100.5)$

$z = \dfrac{64.5 - 80}{6.9282} \approx -2.24$

$z = \dfrac{100.5 - 80}{6.9282} \approx 2.96$

$P(-2.24 \leq Z \leq 2.96) = 0.986$

Example 3: Assume that IQ scores are normally distributed with mean 100 and standard deviation 15. What is the probability that a randomly chosen person will have an IQ at most 105?

given: $\mu = 100$, $\sigma = 15$

Find: $P(X \leq 105)$

$z = \dfrac{105 - 100}{15} \approx 0.33 \xrightarrow{\text{Table}} P(Z \leq 0.33)$

Answer $= 0.6293$

$= 62.93\%$

Example 2: A coin with Pr[Tails] = 0.4 is flipped 200 times. Find the probability of getting between 65 and 100 tails on the coin. Give your answer as a decimal number correct to three decimal places

$$\boxed{\begin{array}{l} n \cdot p > 5 \\ n q > 5 \end{array}}$$

given $P(T) = 0.4$, $n = 200$, $q = 0.6$

Find $P(65 \le X \le 100)$

$* \ \mu = 200(0.4) = 80$

$* \ \sigma = \sqrt{200(0.4)(0.6)} = \sqrt{48} = 6.9282$

$P(64.5 \le X \le 100.5)$

$$z = \frac{64.5 - 80}{6.9282} \approx -2.24 \qquad z = \frac{100.5 - 80}{6.9282} \approx 2.96$$

$P(-2.24 \le Z \le 2.96)$

$= 0.986$

$P(-2.24 \le Z \le 2.96)$

$$= P\left(\frac{65 - 0.5 - 200(0.4)}{\sqrt{200(0.4)(0.6)}} \le Z \le \frac{100 + 0.5 - 200(0.4)}{\sqrt{200(0.4)(0.6)}}\right)$$