Chapter 4

P = ——— / all choices

Section 4.2: Conditional Probability & Independence

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} \qquad \& \qquad P(F \mid E) = \frac{P(E \cap F)}{P(E)}$$

P(E | F): probability of E given F, or probability of E knowing F.
P(F | E): probability of F given E, or probability of F knowing E.

The two events E & F are **independent** if: $P(E \cap F) = P(E).\, P(F)$

Example 1: If P(E) = 2/3, P(F) = 5/8 and P($E \cap F$) = 5/12, are E & F independent?

$\frac{5}{12} \stackrel{?}{=} \frac{2}{3} \cdot \frac{5}{8} \stackrel{?}{=} \frac{10}{24}$ ✓

Example 2: If P(E) = 0.5, P(F) = 0.02 and P($E \cap F$) = 0.2, are E & F independent?

$0.2 \stackrel{?}{=} (0.5)(0.02)$

NO

Example 3: In a survey of 100 people, it was found that:

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	Married (*R*)	Divorced (*D*)	Singles (*S*)	
Male (*M*)	25	7	15	47
Female (*F*)	30	10	13	53
	55	17	28	

If one person is selected, find the probability that this person is:

a) male, female, married, divorced

$P(M) = \frac{47}{100}$, $P(F) = \frac{53}{100}$, $P(R) = \frac{55}{100}$, $P(D) = \frac{17}{100}$

b) male and married, male and divorced, female and married

$P(M \cap R) = 25/100$, $P(M \cap D) = 7/100$, $P(F \cap R) = 30/100$

c) male, given he is married

$$P(M|R) = \frac{P(M \cap R)}{P(R)} = \frac{25/100}{55/100} = \frac{25}{55}$$

First

d) married given the person is female

$$P(R|F) = \frac{P(R \cap F)}{P(F)} = \frac{30/100}{53/100} = \frac{30}{53}$$

e) divorced given the person is male

$$P(D|M) = \frac{7}{47}$$

Example 4: A pair of dice are rolled and the numbers are noted. What is the probability that:

a) both are even given that the sum is 8

$$P(\text{Both even} \mid \text{Sum}=8) = \frac{\text{Both even}}{\text{Sum}=8} = \frac{3}{5}$$

$$5\left\{\begin{array}{l} 44 \\ 53 \\ 35 \\ 62 \\ 26 \end{array}\right.$$

b) the sum is 8 given that both are even.

$$P(\text{Sum}=8 \mid \text{Both are even}) = \frac{\text{Sum}=8}{\text{Both even}} = \frac{3}{9} = \frac{1}{3}$$

22	42	62
24	44	64
26	46	66

Example 5: A box with 7 red balls, 5 white balls and 4 blue balls. 3 are selected at random, find the probability that:

a) they are red given that they are of the same color.

$$P(3R \mid \text{Same Color}) = \frac{3R}{\text{Same Color}} = \frac{C(7,3)}{\underbrace{C(7,3)}_{3R} + \underbrace{C(5,3)}_{3W} + \underbrace{C(4,3)}_{3B}}$$

b) one is white given that at least one is white.

$$P(1W \mid \text{at least } 1W) = \frac{1W + 2\text{ others}}{1W \text{ or } 2W \text{ or } 3W} = \frac{C(5,1)\cdot C(11,2)}{C(5,1)\cdot C(11,2) + C(5,2)\cdot C(11,1) + C(5,3)}$$

$$= \left(\frac{C(5,1)\cdot C(11,2)}{C(16,3) - C(5,0)\cdot C(11,3)}\right)$$

Example 6: There are 7 women and 5 men in a room in which 3 will be selected at random. Find the probability that:

a) all are women given that they are of the same gender.

$$P(3W \mid 3W \text{ or } 3M) = \frac{3W}{3W \text{ or } 3M} = \frac{C(7,3)}{C(7,3) + C(5,3)}$$

b) at least 1 is a man and at least 1 is a woman given that the team contain at least 1 man.

$$P\left(\begin{array}{l}\text{at least } 1M \\ \text{at least } 1W\end{array} \middle| \text{ at least } 1M\right) = \frac{(1M \& 2W) \text{ or } (2M \& 1W)}{(1M \& 2W) \text{ or } (2M \& 1W) \text{ or } (3M)}$$

$$= \frac{C(5,1)\cdot C(7,2) + C(5,2)\cdot C(7,1)}{C(5,1)\cdot C(7,2) + C(5,2)\cdot C(7,1) + C(5,3)}$$

$$= \frac{C(5,1)\cdot C(7,2) + C(5,2)\cdot C(7,1)}{C(12,3) - C(5,0)\cdot C(7,3)}$$

Example 7: A committee consists of 6 Democrats and 5 Republicans. Three of the Democrats are men and three of the Republicans are men. If 2 people are selected, find the probability that they are:

3DM 3RM

3DF 2RF

a) Republican, given they are men.

$$P(2R \mid 2M) = \frac{(2R)\cap M}{2M} = \frac{C(3,2)}{C(6,2)} = 0.2$$

b) opposite gender, given they are Republican.

$$P(1M \& 1F \mid 2R) = \frac{(1M \& 1F)\cap R}{2R} = \frac{C(3,1)\cdot C(2,1)}{C(5,2)} = 0.6$$

Example 8: The probability that Mike will go to college is 0.4 and that he will join the army is 0.5. Find the probability that he will go to either one if: $P(C) = 0.4$, $P(A) = 0.5$

a) the two events are independent.

$$P(C \cup A) = P(C) + P(A) - P(C \cap A) = 0.4 + 0.5 - (0.4)(0.5) = 0.7$$

b) the two events are mutually exclusive.

$$P(C \cup A) = 0.4 + 0.5 - \phi = 0.9$$