Section 4.2: Conditional Probability \& Independence

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P(E \mid F)=\frac{P(E \cap F)}{P(F)} \quad \& \quad P(F \mid E)=\frac{P(E \cap F)}{P(E)}
$$

$\mathrm{P}(E \mid F)$ : probability of $E$ given $F$, or probability of $E$ knowing $F$. $\mathrm{P}(F \mid E)$ : probability of $F$ given $E$, or probability of $F$ knowing $E$.

The two events $E \& F$ are independent if: $\mathrm{P}(E \cap F)=\mathrm{P}(E) . \mathrm{P}(F)$

Example 1: If $\mathrm{P}(E)=2 / 3, \mathrm{P}(F)=5 / 8$ and $\mathrm{P}(E \cap F)=5 / 12$, are $E$ \& $F$ independent?

Example 2: If $\mathrm{P}(E)=0.5, \mathrm{P}(F)=0.02$ and $\mathrm{P}(E \cap F)=0.2$, are $E$ \& $F$ independent?

Example 3: In a survey of 100 people, it was found that:

|  | Married <br> $(R)$ | Divorced <br> $(D)$ | Singles <br> $(S)$ |
| :---: | :---: | :---: | :---: |
| Male $(M)$ | 25 | 7 | 15 |
| Female $(F)$ | 30 | 10 | 13 |

If one person is selected, find the probability that this person is:
a) male, female, married, divorced
b) male and married, male and divorced, female and married
c) male, given he is married
d) married given the person is female
e) divorced given the person is male

Example 4: A pair of dice are rolled and the numbers are noted. What is the probability that:
a) both are even given that the sum is 8
b) the sum is 8 given that both are even.

Example 5: A box with 7 red balls, 5 white balls and 4 blue balls. 3 are selected at random, find the probability that:
a) they are red given that they are of the same color.
b) one is white given that at least one is white.

Example 6: There are 7 women and 5 men in a room in which 3 will be selected at random. Find the probability that:
a) all are women given that they are of the same gender.
b) at least 1 is a man and at least 1 is a woman given that the team contain at least 1 man.

Example 7: A committee consists of 6 Democrats and 5 Republicans. Three of the Democrats are men and three of the Republicans are men. If 2 people are selected, find the probability that they are:
a) Republican, given they are men.
b) opposite gender, given they are Republican.

Example 8: The probability that Mike will go to college is 0.4 and that he will join the army is 0.5 . Find the probability that he will go to either one if:
a) the two events are independent
b) the two events are mutually exclusive.

