Chapter 3

Section 3.3: Combination

Example1: Using the letters A, B and C. How many different 2 letters words can be formed?

Permutation

$\boxed{3 \mid 2} = 6$

$P(3,2) = 6$

AB
BA
AC
CA
BC
CB

6

Example2: Using the names Adam (A), Bob (B) and Carol (C). How many different team of two people can be formed?

Combination

AB
BA

AC
CA

BC
CB

3 diff teams

$\frac{6}{2!} = 3$

Permutation: $P(n, k) = n! / (n - k)!$

Combination: $C(n, k) = n! / (n - k)!.\ k!$

$0! = 1$

$P(5, 2) = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5.4.\cancel{3.2.1}}{\cancel{3!}} = \underbrace{5.4}_{2} = 20$

$C(5, 2) = \frac{5.4}{2} = 10$

$P(5, 1) = 5$

$C(5, 1) = 5$

$P(5, 5) = 5.4.3.2.1 = 5!$

$C(5, 5) = \frac{5.4.3.2.1}{5!} = 1$

$P(5, 0) = 1$

$C(5, 0) = 1$

$P(10, 3) = \underbrace{10.\ 9.\ .\ 8}$

$C(10, 3) = \frac{10.\ 9.\ 8}{3!}$

$C(9, 2) = \frac{9.8}{2!}$

Permutation is when order is important. The process in permutation is: Arranging

Common examples

How many different: words, codes, numbers, ways of seating people, itineraries, ranks, roles…etc**.** In each case,you arrange in certain order.

1 2 3 3 2 1 A B D

D B A

Combination is when order is not important. The process in combination is: Selecting, choosing.

Common examples

How many different: teams of people *(regardless of ranks or roles)*, set of cards, set of courses..etc. In each case, you are selecting regardless of the order or rank.

Again and as we did in Permutation, use the following translations:

When you use the word "Or", then add (+)

When you use the word "And", then Multiply (.)

When you use the word "Except", then Subtract (-)

Example 3: A student must take 4 courses in his school. If there are 5 Math, 4 English, 3 History and 2 Computer courses available. In how many different ways this can be done if:

$\underbrace{\text{5 Math, 4 English, 3 History and 2 Computer courses}}_{\text{14 Courses}}$

a) there is no restriction

4 out of 14 $\longrightarrow$ $C(14,4) = \dfrac{14 \cdot 13 \cdot 12 \cdot 11}{4!} = 1001$

b) he has to take one course of each

1 Math + 1 Eng. + 1 Hist. + 1 Comp.

$C(5,1) \cdot C(4,1) \cdot C(3,1) \cdot C(2,1)$

$5 \cdot 4 \cdot 3 \cdot 2 = 120$

c) any choice must have at least 2 English and at least 1 Math course.

5M, 4E, 3H, 2C $\longrightarrow$ 4 Selected

(2E + 1M + 1 others) or (2E + 2M) or (3E + 1M)

$C(4,2) \cdot C(5,1) \cdot C(5,1) + C(4,2) \cdot C(5,2) + C(4,3) \cdot C(5,1)$

$6 \cdot 5 \cdot \underset{\text{others}}{5} + 6 \cdot 10 + 4 \cdot 5$

$150 + 60 + 20 = 230$

Example 4: A team of 5 people to be selected out of 4 women and 7 men. In how many different ways this can be done if:

11 People
5 selected

a) there is no restrictions

$C(11,5) = 462$

b) the team must have 2 women

$2W + 3M \longrightarrow C(4,2) \cdot C(7,3) = 210$

c) the team must have at least 2 women

2W or more

$(2W+3M)$ or $(3W+2M)$ or $(4W+1M)$

$C(4,2) \cdot C(7,3) + C(4,3) \cdot C(7,2) + C(4,4) \cdot C(7,1)$

$210 + 84 + 7 = 301$

d) the team must have no more than 3 men

3M or less

$(3M+2W)$ or $(2M+3W)$ or $(1M+4W) = 301$

e) the team must have at least 1 woman and at least 3 men

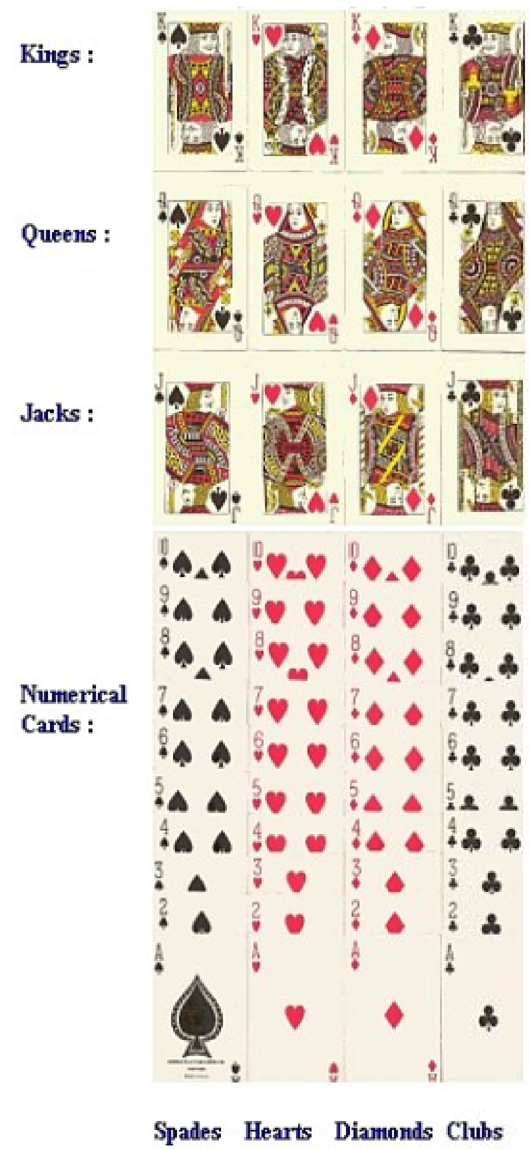
$(1W+4M)$ or $(2W+3M)$ or $(3W + 2M)$

$C(4,1) \cdot C(7,4) + C(4,2) \cdot C(7,3)$

$4 \cdot \frac{7.6.5.4}{4!} + \frac{4.3}{2!} \cdot \frac{7.6.5}{3!}$

$140 + 210 = 350$

<u>Example 5:</u> Five cards to be selected out of 52 cards. How many different ways this can be done if the 5 cards are:



a) any cards (no restrictions)

$C(52, 5)$

b) 3 kings and 2 queen

3K + 2Q

$C(4,3) \cdot C(4,2)$

c) exactly one king

1K + 4 others

$C(4,1) \cdot C(48,4)$

d) same rank

Not Possible

$C(\cancel{4}, 5)$

Example 5 Cont.: Five cards to be selected out of 52 cards. How many different ways this can be done if the 5 cards are:

e) same color

5 Red or 5 Black

$C(26,5) + C(26,5) = 2 \cdot C(26,5)$

2 Colors $\longrightarrow$ $C(2,1) \cdot C(26,5)$

f) more than one color

all except 5R, 5B

$C(52,5) - C(26,5) - C(26,5)$

$C(52,5) - 2 \cdot C(26,5)$

e) same suit

5S or 5H or 5C or 5D

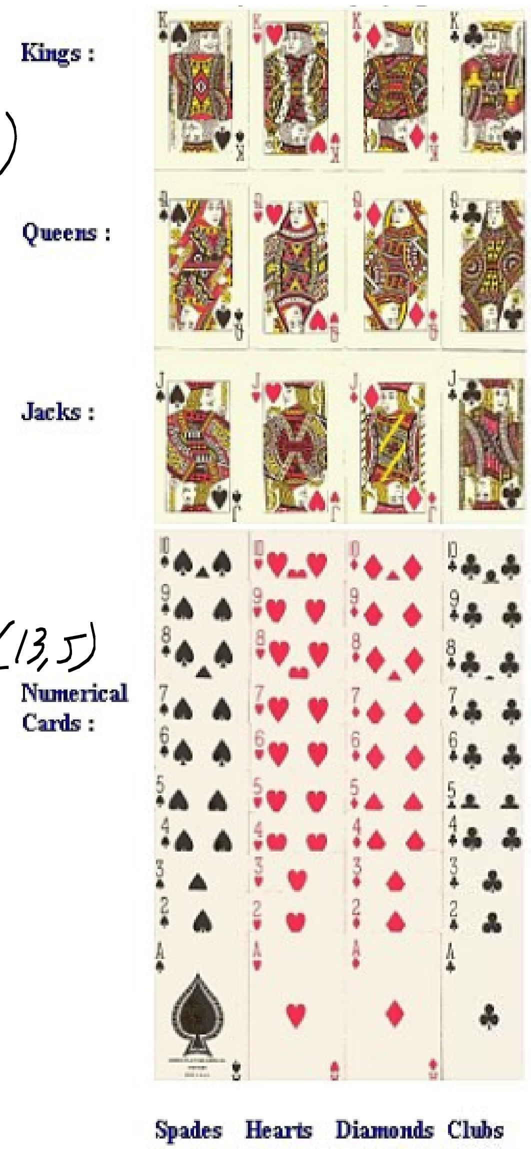
$C(13,5) + C(13,5) + C(13,5) + C(13,5) = 4 \cdot C(13,5)$

$\longrightarrow$ $C(4,1) \cdot C(13,5)$

g) more than one suit

all except Same Suit

$C(52,5) - 4 \cdot C(13,5)$



Example 6: **The Mass lottery involves selecting 6-numbers out-of-46 numbers (1,2,3,4....45,46). In how many ways this can be created:**

6C , 40W

a) getting the correct 6 numbers?

$C(6,6) = 1$

b) getting 4 correct numbers?

4C, 2W

$C(6,4) \cdot C(40,2)$

c) getting 0 correct numbers?

0C , 6W

$C(6,0) \cdot C(40,6) = C(40,6)$

$= 1$

<u>Example 7</u>: In a box there are : 7 red books, 5 white books and 6 blue books. If 4 books are selected , in how many different ways this can be done if:

18 Books → 4 Selected

a) it must include at least 2 white

7R
6B
= 13
others

2W or 3W or 4W

(2W & 2 others) or (3W & 1 other) or (4W)

$C(5,2) \cdot C(13,2) + C(5,3) \cdot C(13,1) + C(5,4)$

$10 \quad 78 + 10 \cdot 13 + 5 = 915$

b) it must include one color

4W or 4R or 4B

$C(5,4) + C(7,4) + C(6,4)$

$5 + 35 + 15 = 55$

c) it must include more than one color

all except Same color

$C(18,4) - \underbrace{C(7,4) - C(5,4) - C(6,4)}$

$3060 - 55 = 3005$

Example 8: **At a party with 12 people, each person shakes hands with everyone else exactly once. How many handshakes have occurred?**

Team of 2

$$C(12,2) = 66$$

Example 9: How many different committees of three can be formed from 12 tennis players and 13 soccer players if at least one tennis player and at least one soccer player must be on the committee?

12T , 13 S $\longrightarrow$ 3 Selected

(1T + 2S) or (2T + 1S)

$$C(12,1)\cdot C(13,2) + C(12,2)\cdot C(13,1) = 1794$$

Example 10 Given a set with 6 elements, how many different subsets containing:

a) exactly 5 elements $\longrightarrow$ $C(6,5)$

b) all possible subsets

0E or 1E or 2E or 3E or 4E or 5E or 6E

$$C(6,0) + C(6,1) + C(6,2) + C(6,3) \ldots + C(6,6)$$

$$\boxed{2^6 = 64}$$

$$\boxed{2^n}$$

Example 11: **12 construction workers to be divided into 3 groups of four each, in how many ways this can be done if:**

a) The first group for **welders**, the second for **concrete** workers, and the third group for **painters.**
(Selecting distinguishable, ordered groups)
(Answer = 34650)

$\boxed{WWWW \mid CCCC \mid PPPP}$

$C(12,4)\cdot C(8,4)\cdot C(4,4) = 34650$

$$\frac{12!}{4!\,4!\,4!} = 34650$$

b) All having same skills **(Selecting undistinguishable, unordered groups)**
(Answer = 5775)

$$\frac{12!}{4!\,4!\,4!\,(3!)} = 5775$$

Example 16: In a conference, 8 managers attended from different divisions and they will be divided into groups of 2 each.

4 Groups

a) In how many ways this selection can be done if the groups are **distinguishable**, ordered?
(Answer = 2520)

$$\frac{8!}{2!\,2!\,2!\,2!} = 2520$$

b) In how many ways this selection can be done if the groups are **undistinguishable**, unordered?
(Answer = 105)

$$\frac{8!}{2!\,2!\,2!\,2!\,(4!)} = 105$$