Chapter 3

Section 3.1, 3.2: Permutation

Example1: How many different 2 letters words can be formed out of the letters A, B and C?

$$3$$

A B
A C
B C   6
B A
C A
C B

| 3 | 2 | = 6 |

Example2: Using the letters A, B, C, D, E and F. How many different words can be formed if the word contains:

a) 3 letters

$$\boxed{6 \mid 5 \mid 4} = 120 \qquad P(6,3) = 6.5.4$$

b) 4 letters

$$\boxed{6 \mid 5 \mid 4 \mid 3} = \boxed{360}$$

$$P(6,4) = 6.5.4.3$$

c) all letters

$$\boxed{6 \mid 5 \mid 4 \mid 3 \mid 2 \mid 1} = 720$$

$$6! \qquad P(6,6) = 6!$$

Factorial

$$P(6,0) = 1$$
$$P(6,1) = 6$$

Factorial Notation: $n! = n.(n-1).(n-2).....2.1$

$$6! = 6.5.4.3.2.1$$

$$!$$

$5! = 5.4.3.2.1 = 120 \qquad\qquad 1! = 1 \qquad\qquad 0! = 1$

Permutation: $P(n, k) = n! / (n - k)!$

$P(5, 2) = \dfrac{5!}{3!} = \dfrac{5.4.3.2.1}{3!} = 20 \qquad\qquad P(5, 2) = \dfrac{5.4}{2} = 20$

$P(5, 1) = \dfrac{5!}{4!} = \dfrac{5.4.3.2.1}{4!} = 5 \qquad\qquad P(5, 1) = 5$

$P(5, 5) = \dfrac{5!}{0!} = \dfrac{5!}{1} = 120 \qquad\qquad P(5, 5) = 5.4.3.2.1 = 5!$

$\underbrace{\qquad}_{5}$

$P(5, 0) = \dfrac{5!}{5!} = \boxed{1} \qquad\qquad P(5, 0) = 1$

$P(10, 3) = \dfrac{10!}{7!} = \dfrac{10.9.8.7.6....1}{7!} \qquad\qquad P(10, 3) = \dfrac{10.9.8}{3} = 720$

$= 720$

$P(7, 3) = \dfrac{7!}{4!} = \dfrac{7.6.5.4.3...1}{4!} \qquad\qquad P(7, 3) = \dfrac{7.6.5}{3}$

Example 3: How many 4-digits number can be formed out of 0,1,2,3,5,7 and 9.

(The question has no restriction, then it is without replacement or each digit can be used only once. Always solve it without replacement, unless the problem specifically asks otherwise)

$$\boxed{7 \mid 6 \mid 5 \mid 4} = 840 \qquad P(7,4) = 840$$

$$0, 1, 2, 3, 5, 7, 9$$

Example 4: How many 4-digits number can be formed out of 0,1,2,3,5,7 and 9, if each digit can be used more than once (with repetition).

$$\boxed{7 \mid 7 \mid 7 \mid 7} = 7^4 = 2401$$

For the next examples, it is easier solve them with those hints:

1) **If there is no restriction, then use the formula**

2) **If there is restriction such as the number must be even, or must start with a certain digit, then solve the restriction first.**

3) **Use the following translations:**

> **When you use the word "Or", then add (+)**
>
> **When you use the word "And", then Multiply (.)**

Example 5: **How many 4-digits number can be formed out of 0,1,2,3,5,7 and 8. If:**
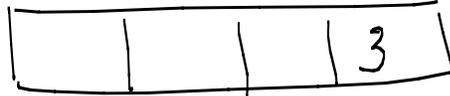
a) **with no repetition (No Restriction)**

$$P(7, 4) = 7 \cdot 6 \cdot 5 \cdot 4 = 840$$

b) **with repetition**

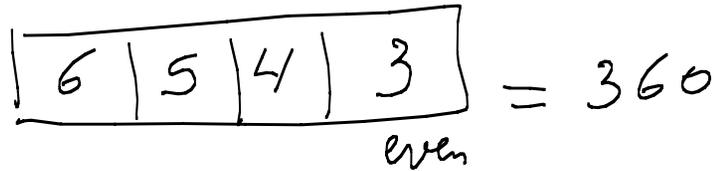$$\boxed{7 \mid 7 \mid 7 \mid 7} = 7^4 = 2401$$

Example 5 Cont.: How many 4-digits number can be formed out of 0,1,2,3,5,7 and 8. If:

c) the number must be even with no repetition, then with repetition

$$\boxed{|||3}$$

↑even 0,2,8

$$\begin{array}{|cccc} 2 & 3 & 5 & 7 \\ \hline 7 & 5 & 3 & 2 \\ & & & = \end{array}$$

With no repetition

$$\boxed{6 | 5 | 4 | 3} = 360$$

even

With repetition

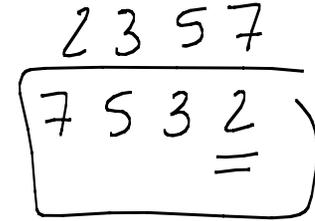$$\boxed{7 | 7 | 7 | 3} = 1029$$

even
0, 2, 8

Example 5 Cont.: How many 4-digits number can be formed out of 0,1,2,3,5,7 and 8. If:

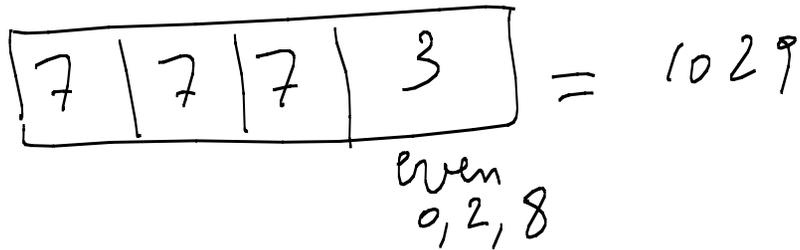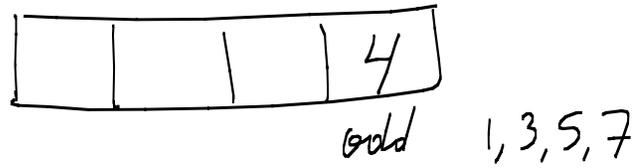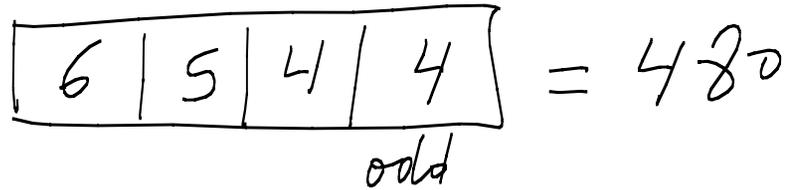d) the number must be <u>odd</u> with no repetition, then with repetition

$$\boxed{ \mid \mid 1 \mid 4}$$

odd 1, 3, 5, 7

With no repetition

$$\boxed{6 \mid 5 \mid 4 \mid 4} = 480$$

odd

With repetition

$$\boxed{7 \mid 7 \mid 7 \mid 4} = 1372 \qquad 2225$$

odd

Example 5 Cont: How many 4-digits number can be formed out of 0,1,2,3,5,7 and 8. If:

e) the number must be larger than 5000 with no repetition

$$3 \mid 6 \mid 5 \mid 4 = 360$$

↑
5 or larger
5, 7, 8

3 5 7 8

5 0 1 2
↑

f) the number must be less than 2000 with no repetition

$$2 \mid 6 \mid 5 \mid 4 = 240$$

↑
0, 1

②0 1 5
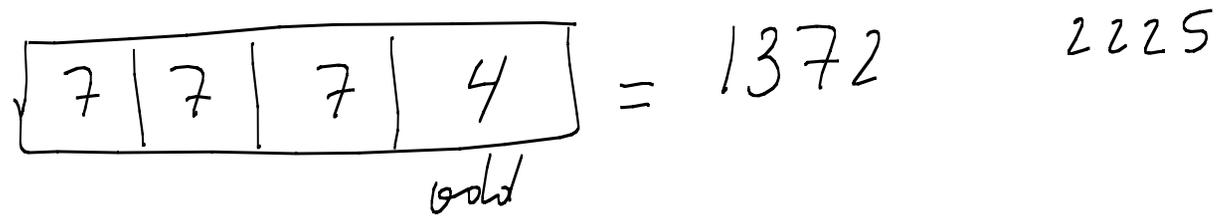
Example 5 Cont: How many 4-digits number can be formed out of 0,1,2,3,5,7 and 8. If:

g) the number must start with 7 or 8 with no repetition

$$\boxed{2 \mid 6 \mid 5 \mid 4} = 240$$

7,8

h) the number cannot start with 7 with no repetition

* *

$$0, 1, 2, \cancel{3}, 5, \textcircled{7}\ 8$$

$$\boxed{6 \mid 6 \mid 5 \mid 4} = 720$$

↑
not 7

Example 5 Cont: How many 4-digits number can be formed out of 0,1,2,3,5,7 and 8. If:

i) the number must have both odd and even digits.

$2\ 3\ 5\ 7$ ✓

$5\ 7\ 3\ 1$ ✗

with no repetition

all except all odds, all even

7	6	5	4	—	4	3	2	1	—		1		1

$1,3,5,7$ $0, 2, 8$

840 $-\ 24\ =\ 816$

with repetition

2008

7	7	7	7	~	4	4	4	4	—	3	3	3	3

$1,3,5,7$ $0,2,8$

7^4 $-\ 4^4$ $-\ 3^4$

$=\ 2064$

Example 6: How many five digit codes can be formed if the first two digits must be non-repeated letters of the alphabet, and the last three digits can be repeated numbers from the set {0 - 9}

Example 7: fair coin is tossed 5 times and the result (heads or tails) is noted on each flip. How many outcomes are in the sample space for this experiment?

Example 8: A certain test has 10 multiple choice questions with 4 choices each, followed by 12 true/false questions. An answer sheet consists of one answer to each question. In how many different ways could you fill in the answer sheet?

Example 9: There are 3 women and 3 men to be seated in a row of 6 chairs.
In how many different ways they can be seated if:

WWW   MMM

a) there is no restriction

$$P(6,6) = 6! = 720$$

b) one woman at each end with no other restrictions

WWW
MMM

3					2

$= 3 \cdot 2 \cdot P(4,4) = 144$

W   No Rest.   W
     P(4,4)

c) they must alternate

WWW
MMM

W M W M W M

3	3	2	2	1	1

$= 36$

or

3	3	2	2	1	1

$= 36$

M   W   M   W   M   W

$36(2) = 72$

$+$

$\dfrac{36}{72}$

d) a particular couple must sit together.

WWW
MMM

			/////
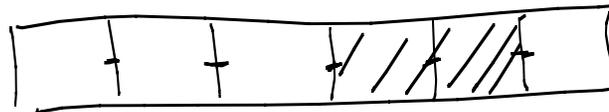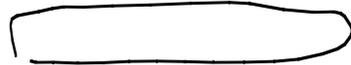
others

$5(2) \cdot P(4,4)$
couple

$= 240$

Example 10: There are 3 women and 3 men in to be seated in a row of 10 chairs. In how many different ways they can be seated if:
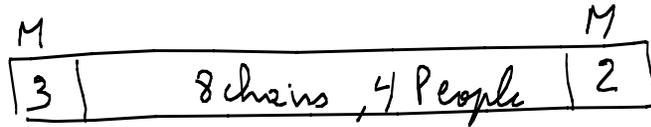
a) there is no restriction

$$P(\cancel{6}/10)$$

$$P(10, 6)$$

b) one man at each end with no other restriction

w w w

M $\cancel{M}$ $\cancel{M}$

M		M
3	8 chairs, 4 People	2

$(3 \cdot 2) \cdot \dfrac{P(8,4)}{\underline{}} = 10080$

Example 11: There are 5 seats numbered 1,2,3,4 and 5 . Seats 1, 2 and 3 for women, seats 4 and 5 for men. There are 5 women and 4 men to be seated, one of the women is Linda. In how many different way they can be seated if:
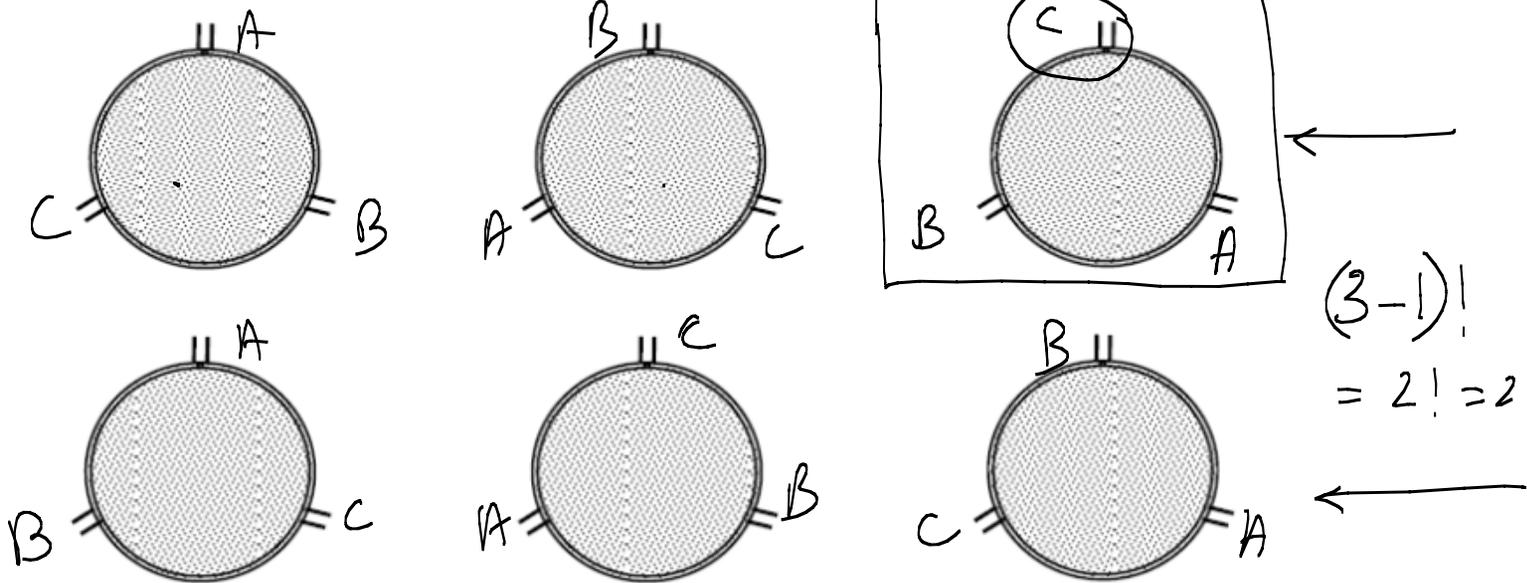
a) there is no restriction

b) if Linda must be included

Example 12a: There are 3 people Adam (A), Bob (B) and Carol (C) to be seated in a row of 3 chairs, in how many different ways they can be seated ?

| 3 | 2 | 1 | $= 6$, $P(3,3) = 6$, $3! = 6$

ABC, ACB, BAC, BCA,
CAB, CBA

Example 12b: Repeat the same question, but to be seated around a (circular) table.

$(3-1)!$
$= 2! = 2$

Example 13: There are 4 women and 4 men in to be seated around a circular table, in how many different ways they can be seated ?

8 People $\longrightarrow$ Row : $8!$

Circular $= (8-1)! = 7!$

Example 14: How many words or numbers can be formed by rearranging;

a) ABCD

$$4! = 24$$

$$\boxed{4 \mid 3 \mid 2 \mid 1} = 24$$

b) ABBA

$$\frac{4!}{2! \; 2!} = 6 \qquad \begin{array}{l} 2B \\ 2A \end{array}$$

c) STATISTICS

3 S
3 T
2 I

$$\frac{10!}{3! \; 3! \; 2!} = 50\,400$$

d) 1112235

$$\bigcirc =$$

$$\frac{7!}{3! \; 2!} = 420$$