Chapter 4

Section 4.1: Probability

Outcomes : a particular result of an experience**.**

Sample Space: the set of all possible outcomes of an experiment.

Example 1: * By rolling a die once, the sample space is $S = (1,2,3,4,5,6)$

* By flipping a coin twice, the sample space is $S = (HH,HT,TH,TT)$

The Probability or (*likelihood*) of the occurrence of an event is:

Probability of an event $P(E) = \frac{n(E)}{n(S)}$

n(E): number of outcomes where the event occurs.
n(S): total number of possible outcomes in the sample space.

Example 2: In a survey of 100 people, it was found that 57 watch the late news.

P(*E*): probability that a person watches late news P(*E*) = 57%
P(*E'*): probability that a person does not watch late news P(*E'*) = 43%

$$P(E) + P(E') = 1$$

Equally Likely: When each of the outcomes of an experiment has the same probability of occurring (*fair die*, *fair coin*....)

Hint: In chapter 3, we were finding: (how many different ways) using permutation or combination. We will use the same concept in probability, but by dividing two numbers:

$$\text{Probability} = \frac{\text{Number of choices of what we are looking for}}{\text{Number of All possible choices}}$$

$$= \frac{\text{Number of All choices (with restriction)}}{\text{Number of All choices (with no restrictions)}}$$

Example 3: A team of 5 people to be selected out of 4 women and 7 men.

a) In **how many different ways** this can be done if there is no restrictions?

b) In **how many different ways** this can be done if the team must have 2 women?

c) What is the **probability** that the team has 2 women?

Example 4: By rolling a pair of dice, in how many different ways the sum is 4, 6, 9, or 12?

Example 5: By rolling a pair of dice, find all outcomes of sums and the probability of (

Sum of	By	# of ways	Prob.
2	11	1	1/36
3	21, 12	2	2/36
4	22, 31, 13	3	3/36
5	32, 23, 41, 14	4	4/36
6	33, 42, 24, 51, 15	5	5/36
7	43, 34, 61,16, 52, 25	6	6/36
8	44, 53, 35, 62, 26	5	5/36
9	54, 45, 63, 36	4	4/36
10	55, 64, 46	3	3/36
11	56, 65	2	2/36
12	66	1	1/36
	Sum =	36	36/36 = 1

Total number of events is 36 or 6x6 = 36

The sum of probabilities for all outcomes is always = 1

Example 6: By rolling a pair of dice, find the probability of:

a) getting the sum of 6.

b) not getting the sum of 6

Example 7: By selecting 5 cards, find the probability of getting:

a) exactly 3 Aces

b) same color

c) same suit

<u>Example 8</u>: In a box there are 15 Science books and 10 History. If 7 books are selected at random (*equally likely*), find the probability of getting at least 1 Science book.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$P(A \cup B)$ = probability of A **or** B (either A or B or both)

$P(A \cap B)$ = probability of both A **and** B

= $P(A).P(B)$; if they are independent *(will be in section 4.2)*

= 0 ; if they are **mutually exclusive** (*disjoint*)

$P(A \cup B)'$ = probability of neither A nor B = $1 - P(A \cup B)$

Example 9: Out of 90 students surveyed, 30 took Math, 40 took English and 10 took both. What is the probability that a student took:

a) English and Math

b) neither English nor Math

Example 10: The probability that Bob will pass the Math course is 0.6, and that he will pass the English course is 0.7. If the probability that he will pass both of them is 0.4, find the probability that:

a) he will pass at least one course.

b) he will not pass any of the courses

c) he will pass either course but not both (only one)

Example 11: A survey in a college found that 40% passed the Math test , 70% passed the English test and 10% passed neither test. What is the probability :

a) of students that passed both test?

b) of students that passed one subject only?

Example 12: Using the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11. If one number is selected, what is the probability that it is less than 4 **or** odd?

<u>The Odds:</u>

- *Given the probability, find the odds*: If the probability of an event E is p, then

Odds <u>for</u> the event $= \dfrac{p}{1-p}$	Odds <u>against</u> the event $= \dfrac{1-p}{p}$

<u>Example 13</u>: The probability for winning a game is P(W) = 7/12. What is the odds:

a) for winning

b) against winning

- *Given the odds, find the probability*: If the odds for making an event E are a to b, then:

Probability of $(E) = \dfrac{a}{a+b}$	Probability of $(E') = \dfrac{b}{a+b}$

<u>Example 14:</u> The odds for winning a game is 7/5. What is the probability of:

a) winning

b) loosing

<u>Section 4.2:</u> Conditional Probability & Independence

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} \quad \& \quad P(F \mid E) = \frac{P(E \cap F)}{P(E)}$$

P(E | F): probability of E given F, or probability of E knowing F.
P(F | E): probability of F given E, or probability of F knowing E.

The two events E & F are **independent** if: $P(E \cap F) = P(E).\, P(F)$

<u>Example 1:</u> If $P(E) = 2/3$, $P(F) = 5/8$ and $P(E \cap F) = 5/12$, are E & F independent?

<u>Example 2:</u> If $P(E) = 0.5$, $P(F) = 0.02$ and $P(E \cap F) = 0.2$, are E & F independent?

Example 3: In a survey of 100 people, it was found that:

	Married (R)	Divorced (D)	Singles (S)	
Male (M)	25	7	15	
Female (F)	30	10	13	

If one person is selected, find the probability that this person is:

a) male, female, married, divorced

b) male and married, male and divorced, female and married

c) male, given he is married

d) married given the person is female

e) divorced given the person is male

Example 4: A pair of dice are rolled and the numbers are noted. What is the probability that:

a) both are even given that the sum is 8

b) the sum is 8 given that both are even.

Example 5: A box with 7 red balls, 5 white balls and 4 blue balls. 3 are selected at random, find the probability that:

a) they are red given that they are of the same color.

b) one is white given that at least one is white.

<u>Example 6:</u> There are 7 women and 5 men in a room in which 3 will be selected at random. Find the probability that:

a) all are women given that they are of the same gender.

b) at least 1 is a man and at least 1 is a woman given that the team contain at least 1 man.

Example 7: A committee consists of 6 Democrats and 5 Republicans. Three of the Democrats are men and three of the Republicans are men. If 2 people are selected, find the probability that they are:

a) Republican, given they are men.

b) opposite gender, given they are Republican.

Example 8: The probability that Mike will go to college is 0.4 and that he will join the army is 0.5. Find the probability that he will go to either one if:

a) the two events are independent

b) the two events are mutually exclusive.

Section 4.3: Bayes Theorem

Use the tree method when you have an experiment which consists of a sequence of sub-experiments.

Example 1: Two people will be selected without replacement out of 7 women and 2 men.

a) draw the tree and show all the probabilities.
b) find the probability that 2 women are selected
c) find the probability that 2 of the same gender are selected

NOTE: By using the tree method:

a) the sum of all path probabilities must be = 1
b) the sum of probabilities of all branches from one node = 1
c) the sum of all path probabilities that branches from a given node must be equal to the probability reaching that node.

Example 2: Repeat example 1 but this time 3 people are selected without replacement, and find the probability of:

a) exactly 2 are women

b) at least 2 are women

c) 1 man given that at least 2 are women

Example 3: At a state university, 60% are undergraduates, 35% graduates and 5% are in special program. Also, 20% of the undergraduates are married, 40% of the graduates are married and 70% of the special program are married. Draw the tree and find the following probabilities that:

a) a selected student is married and undergraduate

b) a selected student is married

c) a selected married student is an undergraduate

Example 4: A fair coin is flipped until 2 heads or 3 tails appear. Draw a tree and determine all probabilites

Example 5: A box contains 10 good parts and 3 defective parts, if parts are selected without replacement one after another until either 2 defective parts are found or three are selected. Draw the tree and show the probabilities

Example 6: In a certain class, there were 10% freshman, 30% sophomores, 40% juniors and 20% seniors. Past experiences show that 20% of freshmen , 40% of sophomores, 30% of juniors and 10% of seniors get A. If one student was selected at random:

a) find the probability that this student got an A

b) if the student found to be an A student, find the probability that this student was a junior

Example 7: Two groups of students applied for a job, graduate group (4 women and 6 men) and undergraduate group (3 women and 5 men). The company would flip an unfair coin in which P(H) = 2/3, if it is a head then the graduate group will be selected and a student from that group will be selected.

a) find the probability that a man is selected

b) if the person selected was a man, find the probability that he is from the undergraduate group

Example 8: Box A contains 4 white books and 6 red books.
Box B contains 3 white books and 2 red books.
Box C contains 2 white books and 4 red books.

a) if a box was selected and <u>then</u> a book was selected, what is the probability that this book is white

b) if the book selected was white, what is the probability that this book was from box B?

<u>Example 9:</u> An airline company is planning to screen all employees for the use of illegal drugs. The test has two results positive or negative. Positive result indicates that illegal drugs were used while negative result indicates no illegal drugs were used. The lab that is doing the test found from previous record that:

If a person did use illegal drugs, the test will detect is (show positive result) in 96% of the cases
(*this means: a **<u>false negative</u>** of 4% because it shows negative in 4% even though the person is a drug user)*

If a person did not use illegal drugs, the test will still show positive result in 10% of the cases
*(this means a **<u>false positive</u>** of 10%)*

At the end of the test, it was found that 5% of the employee did **<u>actually</u>** used illegal drugs. What is the probability:

a) that a person who tests positive actually did use illegal drugs?
b) that a person who actually use illegal drugs tests negative ?
c) that a person who tests negative actually did not use illegal drugs?
d) that a person who actually did not use illegal drugs tests positive ?

Section 4.4: Bernoulli Trials

Example 1: The probability that a team will win a game is 60%. Find the probability that:

a) the team wins a game out of 2

b) the team wins the first 2 games out of 3

c) the team wins 2 game out of 3

d) the team wins 2 games out of 4

Bernoulli trial: (repeated events) is applied when:

1) each event has two outcomes only, (win, loose); (pass, fail)...

2) the sum of the two probabilities for the two outcomes is = 1

3) the events are independent

4) the probability in the repeated events is the same

$$\mathbf{P = C(}\boldsymbol{n}\mathbf{,}\ \boldsymbol{r}\mathbf{)\ .\ }\boldsymbol{p^{r}}\mathbf{.\ }\boldsymbol{q^{n-r}} \qquad \mathbf{(}\boldsymbol{q}\mathbf{\ = 1 - }\boldsymbol{p}\mathbf{)}$$

p : probability of success *(what we are looking for)*

n : total number of trials

r : number of successes *(number of events of what we are looking for)*

Example 2: the probability of winning a game is 60%. If the team plays 8 games, find the probability that the team wins:

a) 5 games   b) at least 6 games   c) at least 2 games

Example 3: By taking a test of 10 questions, each question has 4 choices for an answer and only one answer is correct. If a student is answering the questions by guessing, find the probability that he gets at least 2 correct questions