Chapter 3

Section 3.1: Trees

Example 1: **Two people will be selected without replacement out of 7 women and 2 men, draw the tree and show all the possibilities. What is the sample space?**

<u>Example 2:</u> : Repeat *Example 1* but this time 3 people are selected without replacement out of 7 women and 2 men, draw the tree and show all the possibilities. What is the sample space?

<u>Example 3:</u> : A fair coin is flipped until 1 head occur or 3 flips. How many outcomes in the sample space?

Example 4: A box contains 10 good parts and 3 defective parts, if parts are selected without replacement one after another until either 2 defective parts are found or three are selected. Draw the tree and show all outcomes

Example 5: **A box contains 1 red, 1 white and 2 green balls. An experiment consists of drawing balls in succession without replacement, and noting the color of each until a red ball is drawn. Draw the tree diagram and find how many outcomes in the sample space.**

Example 6: **Suppose you have $50 to spend on meals and that a meal in an expensive restaurant (E) costs $20 and a meal in a moderate priced restaurant (M) costs $10. An experiment consists of deciding on a sequence of meals (expensive or moderate) whose total cost is exactly $50.**
Draw a tree diagram and find the sample space.

Section 3.1, 3.2: Permutation

Example1: How many different 2 letters words can be formed out of the letters A, B and C?

Example2: Using the letters A, B, C, D, E and F. How many different words can be formed if the word contains:

a) 3 letters

b) 4 letters

c) all letters

Factorial Notation: $n! = n.(n - 1).(n - 2).....2.1$

$5! = 5.4.3.2.1 = 120$ $1! = 1$ **$0! = 1$**

Permutation: $P(n, k) = n! / (n - k)!$

$P(5 , 2) =$

$P(5 , 1) =$

$P(5 , 5) =$

$P(5 , 0) =$

$P(10, 3) =$

Example 3: **How many 4-digits number can be formed out of 0,1,2,3,5,7 and 9.**

(The question has no restriction, then it is without replacement or each digit can be used only once. Always solve it without replacement, unless the problem specifically asks otherwise)

Example 4: **How many 4-digits number can be formed out of 0,1,2,3,5,7 and 9, if each digit can be used more than once (with repetition).**

For the next examples, it is easier solve them with those hints:

1) **If there is no restriction, then use the formula**

2) **If there is restriction such as the number must be even, or must start with a certain digit, then solve the restriction first.**

3) **Use the following translations:**

   When you use the word "Or", then add (+)

   When you use the word "And", then Multiply (.)

<u>Example 5:</u> How many 4-digits number can be formed out of 0,1,2,3,5,7 and 8. If:

a) with no repetition (No Restriction)

b) with repetition

Example 5 Cont.: **How many 4-digits number can be formed out of 0,1,2,3,5,7 and 8. If:**

c) the number must be even with no repetition, then with repetition

With no repetition

With repetition

Example 5 Cont.: **How many 4-digits number can be formed out of 0,1,2,3,5,7 and 8. If:**

d) the number must be odd with no repetition, then with repetition

With no repetition

With repetition

Example 5 Cont: **How many 4-digits number can be formed out of 0,1,2,3,5,7 and 8. If:**

e) the number must be larger than 5000 with no repetition

f) the number must be less than 2000 with no repetition

Example 5 Cont: **How many 4-digits number can be formed out of 0,1,2,3,5,7 and 8. If:**

g) the number must start with 7 or 8 with no repetition

h) the number cannot start with 7 with no repetition

<u>Example 5 Cont:</u> How many 4-digits number can be formed out of 0,1,2,3,5,7 and 8. If:

i) the number must have both odd and even digits.

with no repetition

with repetition

Example 6: **How many five digit codes can be formed if the first two digits must be non-repeated letters of the alphabet, and the last three digits can be repeated numbers from the set {0 - 9}**

Example 7: **fair coin is tossed 5 times and the result (heads or tails) is noted on each flip. How many outcomes are in the sample space for this experiment?**

Example 8: **A certain test has 10 multiple choice questions with 4 choices each, followed by 12 true/false questions. An answer sheet consists of one answer to each question. In how many different ways could you fill in the answer sheet?**

<u>Example 9:</u> There are 3 women and 3 men to be seated in a row of 6 chairs.
In how many different ways they can be seated if:

a) there is no restriction

b) one woman at each end with no other restrictions

c) they must alternate

d) a particular couple must sit together.

Example 10: **There are 3 women and 3 men in to be seated in a row of 10 chairs. In how many different ways they can be seated if:**

a) there is no restriction

b) one man at each end with no other restriction

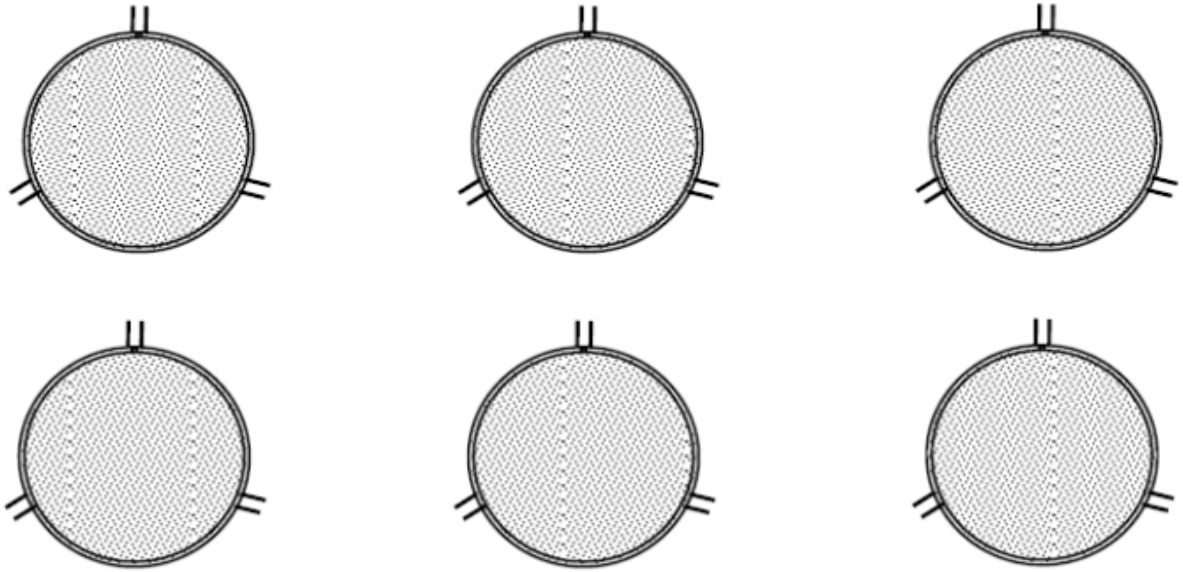
Example 11: **There are 5 seats numbered 1,2,3,4 and 5 . Seats 1, 2 and 3 for women, seats 4 and 5 for men. There are 5 women and 4 men to be seated, one of the women is Linda. In how many different way they can be seated if:**

a) there is no restriction

b) if Linda must be included

Example 12a: **There are 3 people Adam (A), Bob (B) and Carol (C) to be seated in a row of 3 chairs, in how many different ways they can be seated ?**

Example 12b: Repeat the same question, but to be seated around a circular table.



Example 13: **There are 4 women and 4 men in to be seated around a circular table, in how many different ways they can be seated ?**

Example 14: **How many words or numbers can be formed by rearranging;**

a) ABCD

b) ABBA

c) STATISTICS

d) 1112235

Section 3.3: Combination

Example1: Using the letters A, B and C. How many different 2 letters words can be formed?

Example2: Using the names Adam (A), Bob (B) and Carol (C). How many different team of two people can be formed?

Permutation: $P(n, k) = n! / (n - k)!$

Combination: $C(n, k) = n! / (n - k)!. k!$

$P(5 , 2) =$

$P(5 , 1) =$

$P(5 , 5) =$

$P(5 , 0) =$

$P(10, 3) =$

$C(5 , 2) =$

$C(5 , 1) =$

$C(5 , 5) =$

$C(5 , 0) =$

$C(10, 3) =$

Permutation is when order is important. The process in permutation is: Arranging

Common examples
How many different: words, codes, numbers, ways of seating people, itineraries, ranks, roles…etc**.** In each case,you arrange in certain order.

Combination is when order is not important. The process in combination is: Selecting, choosing.

Common examples
How many different: teams of people *(regardless of ranks or roles)*, set of cards, set of courses..etc. In each case, you are selecting regardless of the order or rank.

Again and as we did in Permutation, use the following translations:

When you use the word "Or", then add (+)

When you use the word "And", then Multiply (.)

When you use the word "Except", then Subtract (-)

<u>Example 3:</u> A student must take 4 courses in his school. If there are 5 Math, 4 English, 3 History and 2 Computer courses available. In how many different ways this can be done if:

a) there is no restriction

b) he has to take one course of each

c) any choice must have at least 2 English and at least 1 Math course.

<u>Example 4:</u> A team of 5 people to be selected out of 4 women and 7 men. In how many different ways this can be done if:

a) there is no restrictions

b) the team must have 2 women

c) the team must have at least 2 women

d) the team must have no more than 3 men

e) the team must have at least 1 woman and at least 3 men

<u>Example 5:</u> Five cards to be selected out of 52 cards. How many different ways this can be done if the 5 cards are:

a) any cards (no restrictions)

b) 3 kings and 2 queen

c) exactly one king

d) same rank

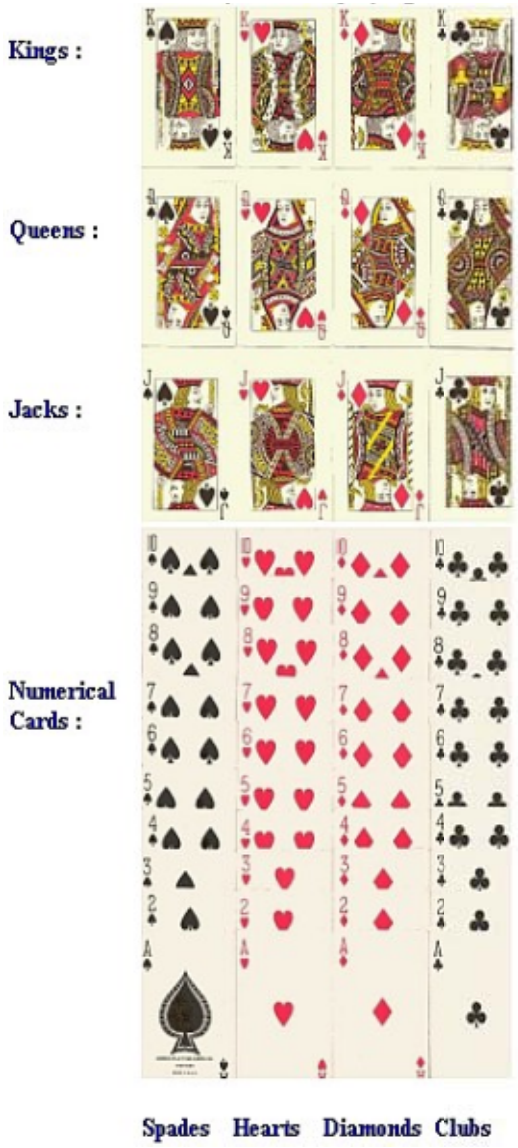
Example 5 Cont.: **Five cards to be selected out of 52 cards. How many different ways this can be done if the 5 cards are:**

e) same color

f) more than one color

e) same suit

g) more than one suit



<u>Example 6:</u> The Mass lottery involves selecting 6-numbers out-of-46 numbers (1,2,3,4....45,46). In how many ways this can be created:

a) getting the correct 6 numbers?

b) getting 4 correct numbers?

c) getting 0 correct numbers?

Example 7: In a box there are : 7 red books, 5 white books and 6 blue books. If 4 books are selected , in how many different ways this can be done if:

a) it must include at least 2 white

b) it must include one color

c) it must include more than one color

Example 8: **At a party with 12 people, each person shakes hands with everyone else exactly once. How many handshakes have occurred?**

Example 9: **How many different committees of three can be formed from 12 tennis players and 13 soccer players if at least one tennis player and at least one soccer player must be on the committee?**

Example 10 **Given a set with 6 elements, how many different subsets containing:**

a) exactly 5 elements

b) all possible subsets

Example 11: **12 construction workers to be divided into 3 groups of four each, in how many ways this can be done if:**

a) The first group for **welders**, the second for **concrete** workers, and the third group for **painters.**
(Selecting distinguishable, ordered groups)
(Answer = 34650)

b) All having same skills **(Selecting undistinguishable, unordered groups)**
(Answer = 5775)

Example 16: In a conference, 8 managers attended from different divisions and they will be divided into groups of 2 each.

a) In how many ways this selection can be done if the groups are **distinguishable**, ordered?
(Answer = 2520)

b) In how many ways this selection can be done if the groups are **undistinguishable**, unordered?
(Answer = 105)