Chapter 2 Set Theory

Section 2.1: Sets and Subsets

A set is a collection of items, referred to as the elements of the set.

Example 1: A **= Northwest States =** $\{WI, MN, ND, MT, ID, WA\}$

The set represent group of states in which each state is an element that is included in the set.

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$ID \in A$; also $MN, ND \in A$

But $IN \notin A$ *(Indiana is not an element of set A)*

Example 2: **If** $A = \{a, c, d, e, f\}$ **and**

$B = \{b, c, d\}$; $C = \{a, b, d\}$; $D = \{a, b, d, g\}$

$B \subseteq A$; B **<u>contained</u>** by A, or B is **<u>subset</u>** of A.

(each element of B is included in A)

$C \subseteq A$; but $D \nsubseteq A$ because g is not included in A

Important note: which of the following is correct and why?:

a) $b, c \in A$
b) $b, c \subseteq A$
c) $\{b, c\} \subseteq A$
d) $\{b, c\} \in A$

Set-Builder Notation:

Example 3: **I = { x | x is an integer between 2 and 8} = {2, 3, 4, 5, 6, 7, 8}.**
The vertical line | is read "such as"

Example 4: **I = { x | x is even and $1 < x < 10$} = {2, 4, 6, 8}**

of Subsets:

Example 5: **If** ***A* = { *A*, *B*}** **;** **(*Art and Biology)***
How many decisions can be made regarding taking any of the above courses?

Example 6: **If** ***A* = { *A*, *B*, *C*}** **;** **(*Art, Biology and Computer)***
How many decisions can be made regarding taking any of the above courses?

# of elements	# of subsets	Example	Subsets
1	2	**A** = {a}	{a}, {∅}
2	4	**A** = {a, b}	{a}, {b}, {a, b}, {∅}
3	8	**A** = {a, b, c}	{a}, {b}, {c} , {a, b}, {a, c}, {b, c}, {a, b, c}, {∅}
4			
5			
6			
7			
8			
n			

<u>Cardinality</u>: the number of elements in a set.

Example 7: **If $A = \{a, b, c\}$; $n(A) = 3$**

<u>Universal Set *U*</u>: The overall set where all other sets are substes of it.

Example 8: *U* = {IUPUI students} with the following subsets:

B = { Business students}
F = {Fresmen students}
R = {Resident students}
S = {Senior students}

All of the above are substets of the universal set *U*.

<u>Complement of a set:</u> ***(*what is missing from a subset compared to the universal set*)***

Example 9: $U = \{a, b, c, d, e, f, g, h\}$; $A = \{a, c, f\}$, $B = \{b, c, g, h\}$

Both sets *A* and *B* are subsets of the universal set *U* where:

$A' = \{b, d, e, g, h\}$ **, the elements missing from *A***

$B' = \{a, d, e, f\}$ **, the elements missing from *B***

Section 2.2: Set Operatrion

Example 1: Let $U = \{ a, b, c, d, e, f, g, h, i\}$ with the following subsets

$A = \{a, b, d, e\}$, $B = \{b, c, e, f, g\}$, $C = \{e, f, h, i\}$

Find the following:

a) A'

b) B'

c) $A \cup B$: The **union** of *A* and *B* is the set of all elements that are in *A* **or** *B* *(or both)*

d) $A \cap B$: The **intersection** of *A* and *B* is the set of all elements that are in *A* **and** *B*.

e) $A \cap (B \cup C)$

f) $(A \cap B) \cup C)$

Example 1 Cont.: Let $U = \{ a, b, c, d, e, f, g, h, i\}$ with the following subsets

$A = \{a, b, d, e\}$, $B = \{b, c, e, f, g\}$, $C = \{e, f, h, i\}$

g) $(A - B)$: What is in A and not in B

h) $(B - A)$: What is in B and not in A

i) $(U - A)$: What is U and not in A, which is the same as A'

Example 2: If $A = \{1, 2, 3\}$, $B = \{5, 6, 7\}$, $C = \{2, 4\}$

Find the following

a) $A \cup B$:

b) $A \cap B$:

c) $A - B$

d) A x C (**Cartesian product**)

e) C x A

Section 2.3: Venn Diagram

Example 3: If $U = \{a, b, c, d, e, f, g, h, i\}$ and $A = \{a, b, c, f\}$, $B = \{b, c, d, e, g\}$ Find:

1) A' ; B'

2) $A \cup B$

$(A \cup B)'$

3) $A \cap B$

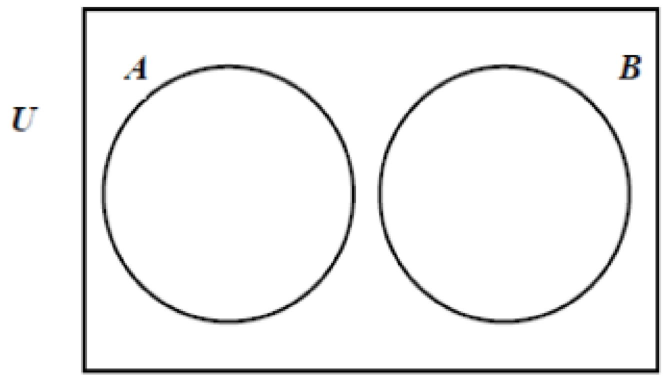
$(A \cap B)'$

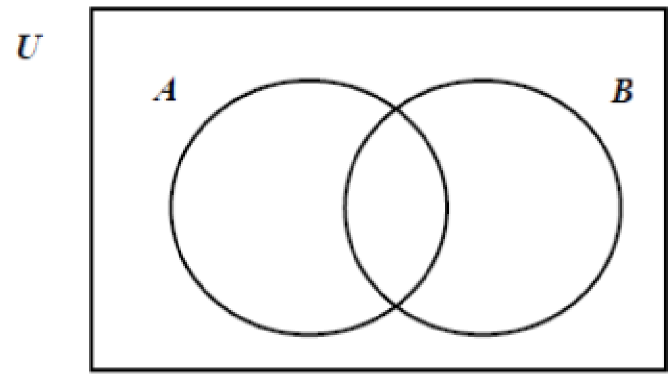
4) $A' \cap B'$

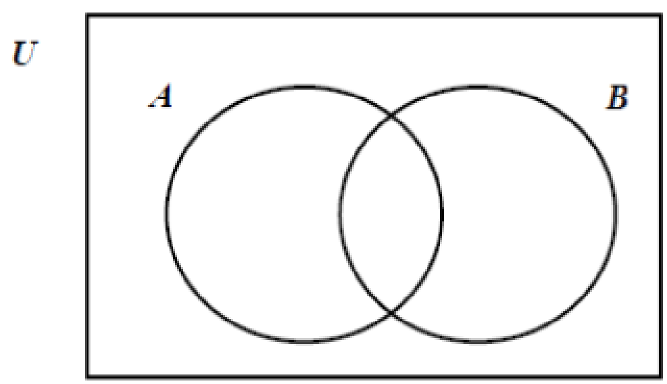
5) $A' \cup B'$

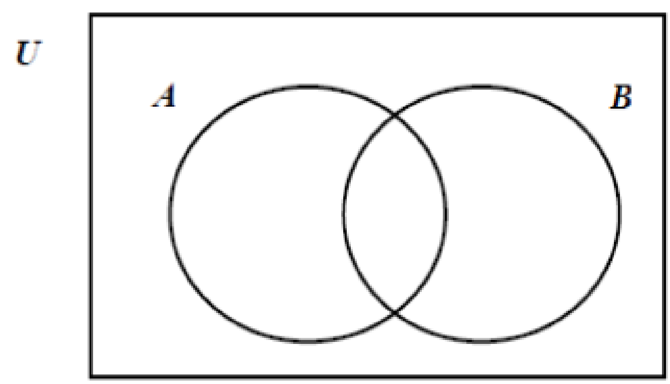
De Morgan Law:	a) $(A \cup B)' = A' \cap B'$
	b) $A' \cup B' = (A \cap B)'$

Example 3 Cont.: If $U = \{a, b, c, d, e, f, g, h, i\}$ and $A = \{a, b, c, f\}$, $B = \{b, c, d, e, g\}$. Draw the Venn diagram









Example 4: If $U = \{a, b, c, d, e, f, g\}$ and $A = \{a, b, f\}$, $B = \{c, d, e, g\}$ Find:

1) $A \bigcup B$

2) $A \bigcap B$

Partition: a) Union is all or: $A \bigcup B = U$ b) Nothing in Common or: $A \bigcap B = \phi$

Example 5: Mark has two sets of courses to choose form:

Set A = {Chemistry, Math, English} = $\{C,M,E\}$

Set B = {French, History, Geology} = $\{F,H,G\}$

Find:

a) the number of courses that are in *A* <u>and</u> *B*.

b) the number of courses that are in *A* <u>or</u> *B*.

Example 6: Mike has two sets of courses to choose form:

Set A = {Chemistry, Math, English, History} = $\{C, M, E, H\}$

Set B = {Math, English, French} = $\{M, E, F\}$

Find:

a) the number of courses that are in A <u>and</u> B. ☛ $n(A \cap B)$

b) the number of courses that are in A <u>or</u> B. ☛ $n(A \cup \text{B})$

c) the number of courses that are in A only.

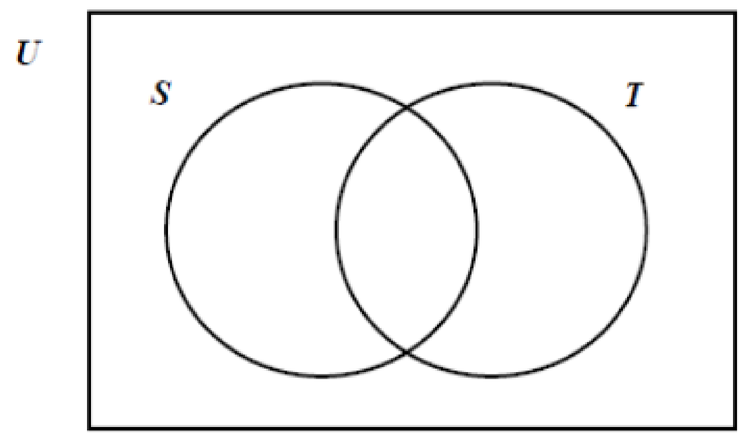
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Example 7: In a survey of 80 people, it was found that:

45 read the Sport magazine (*S*)
40 read the Time magazine (*T*)
10 read both magazines (*T & S*)

Find the number of people that read:

a) Time only **b) Sport only** **c) neither magazine** **d) either magazine**

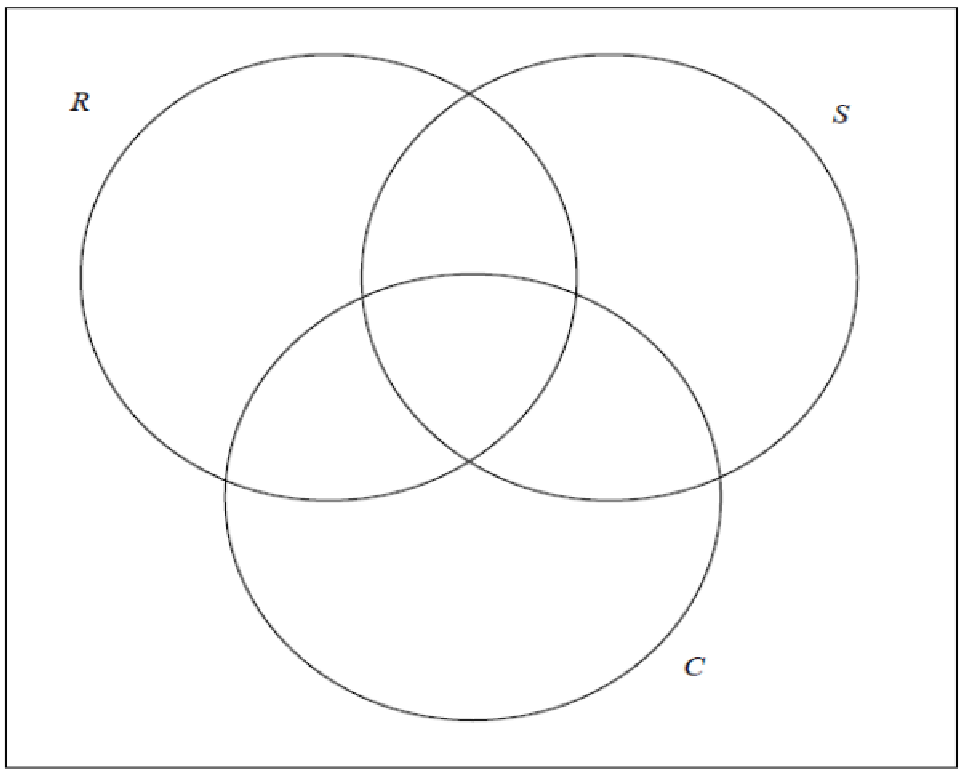


Example 8: **In a survey of 200 people, it was found that:**

150 listen to Rock music (*R*)
80 listen to Slow music (*S*)
55 listen to Classic music (*C*)
60 listen to Rock and Slow music (*R & S*)
25 listen to Classic and Slow music (*C & S*)
40 listen to Rock and Classic (*R & C*)
15 listen to all (*R & S & C*)

Find the number of people that listen to:

a) Rock only **b) 2 kind of music** **c) Rock and Slow but not Classic** **d) none**

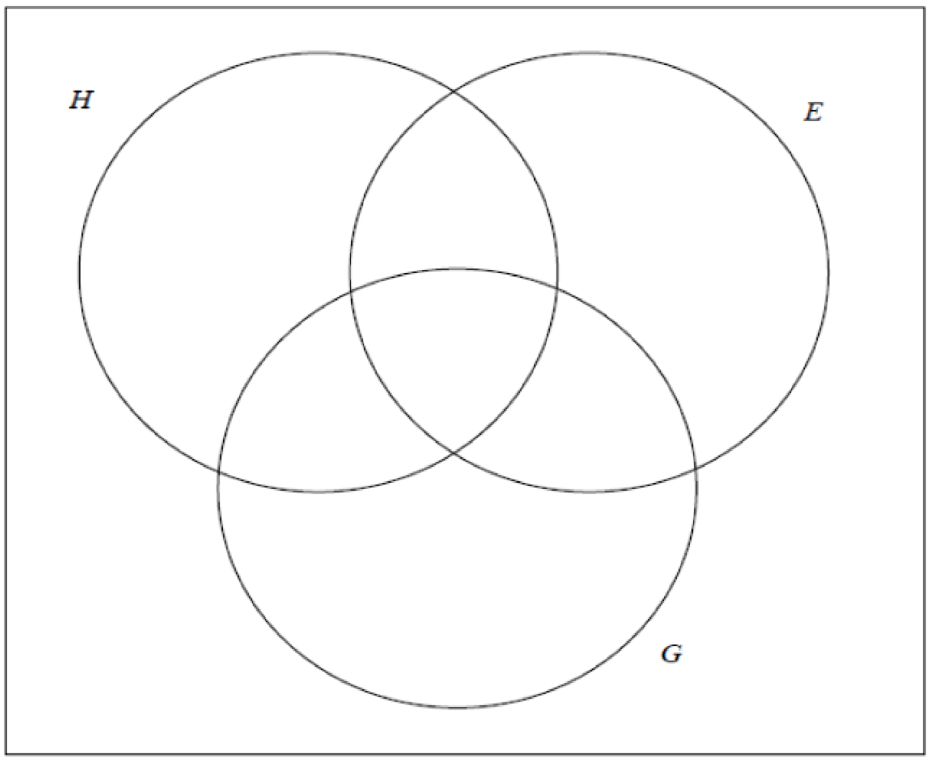


Example 9: In a survey , it was found that:

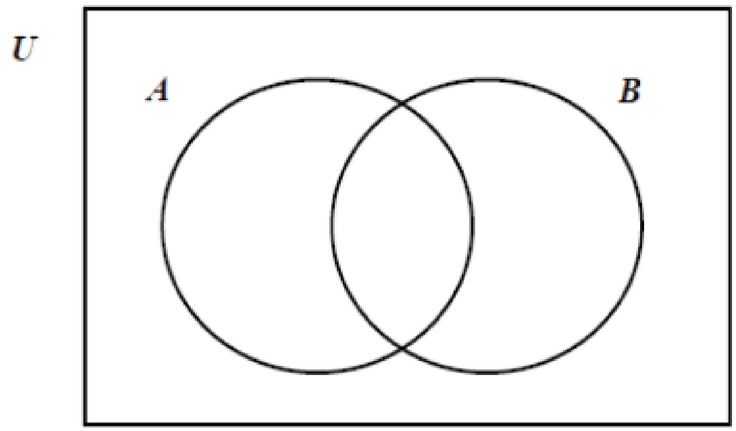
55 students took History (*H*)
45 students took English (*E*)
25 students took Geography (*G*)
7 students took English and History but not Geography
~~5~~ students took Geography and History but not English
3 students took Geography and English but not History
30 students took English only

Find the number of students that took:

a) the three subjects at the same time b) History only

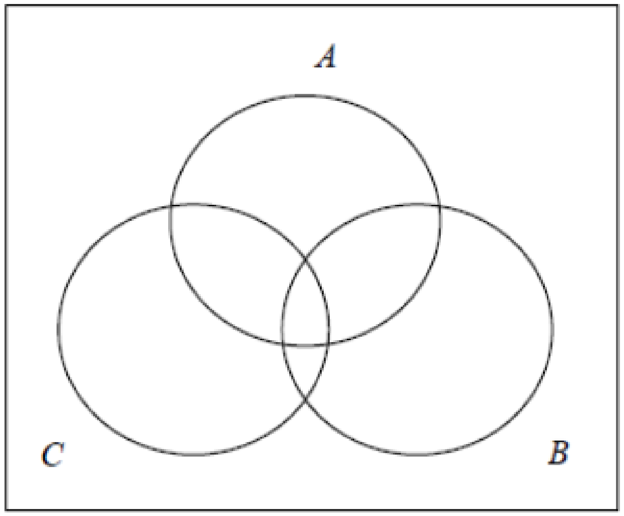


Example 10: If A and B are subsets of U and: $n(A) = 5$, $n(B') = 7$, $n(A' \cap B') = 3$. Find $n(A \cap B)$.

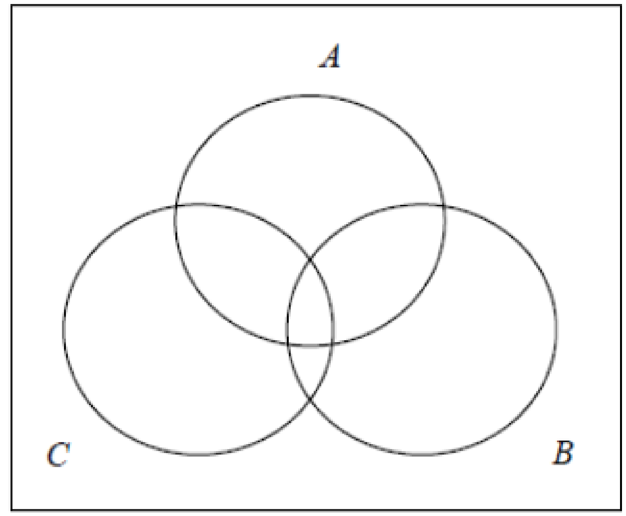


Example 11: Let *A*, *B*, and *C* be subsets of *U*, use the Venn diagram to shade the solution:

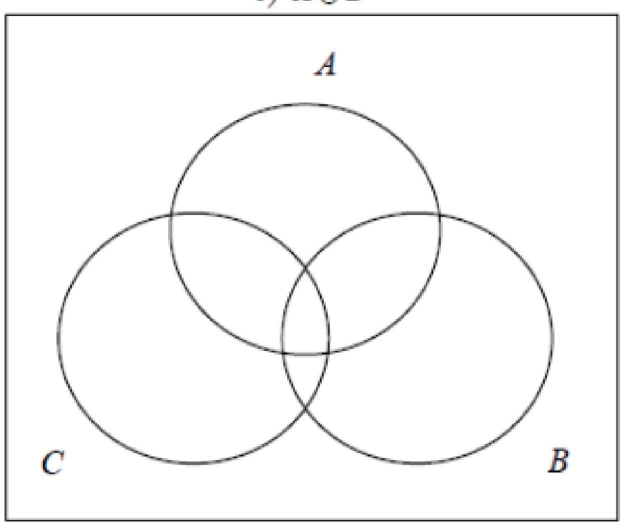
a) $A \cap B \cap C$



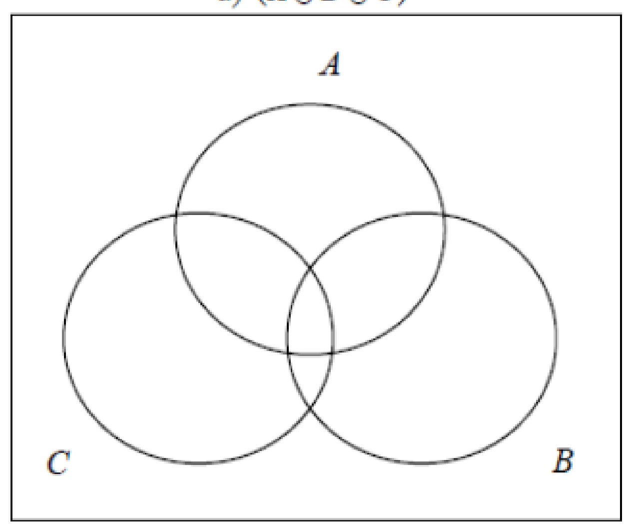
b) $A \cap B$



c) $A \cup B$

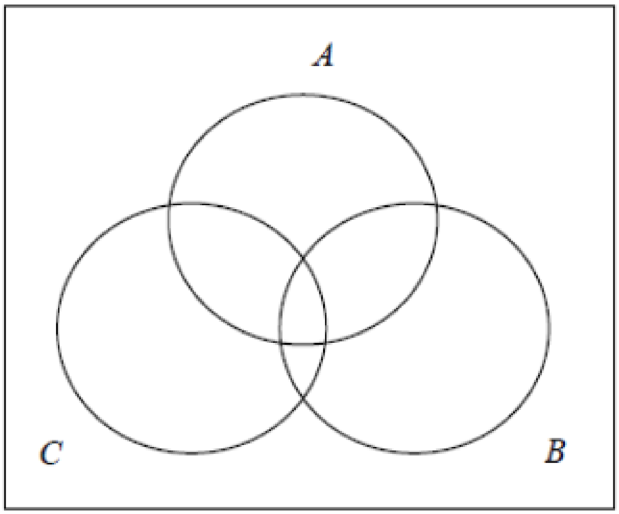


d) $(A \cup B \cup C)'$

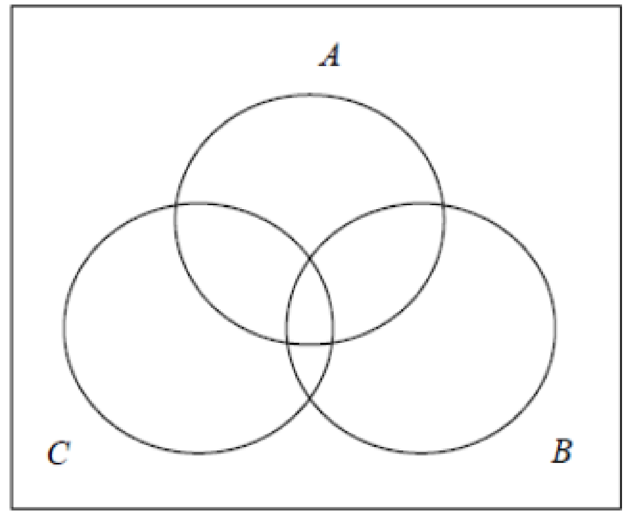


Example 11 Cont.: Let *A, B,* and *C* be subsets of *U*, use the Venn diagram to shade the solution:

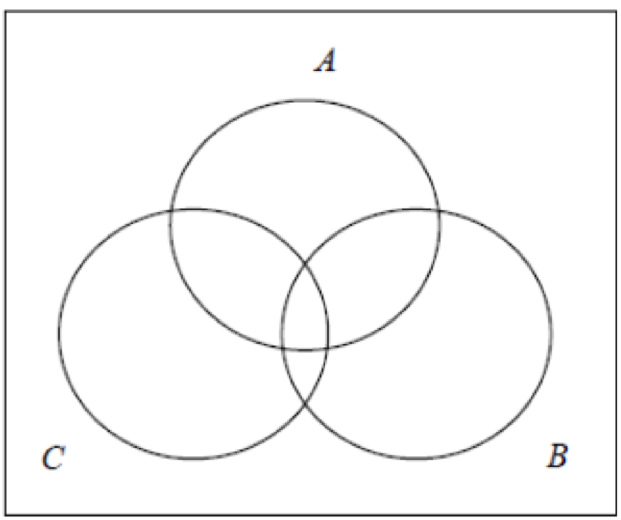
e) $A \cap B'$



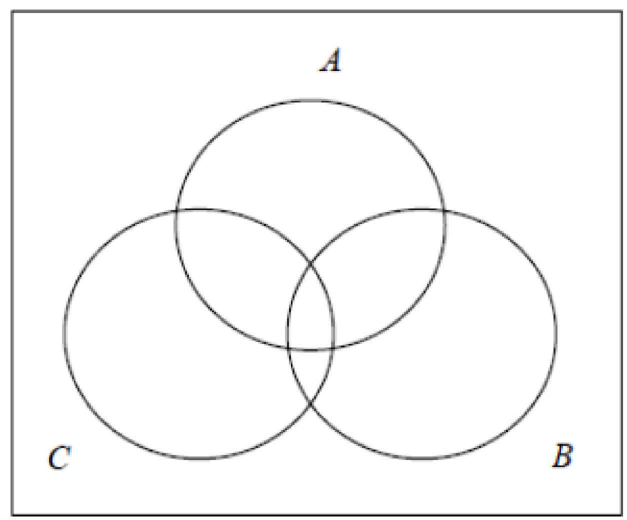
f) $(A \cap B) \cap C'$



g) $(A \cup B) \cap C'$

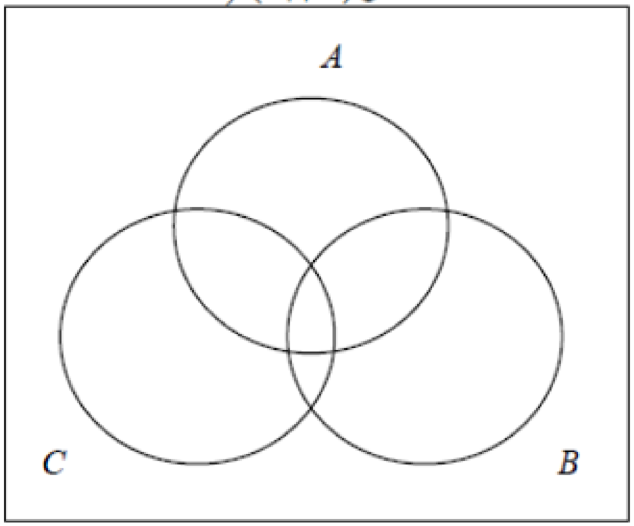


h) $A' \cap (B \cap C)$

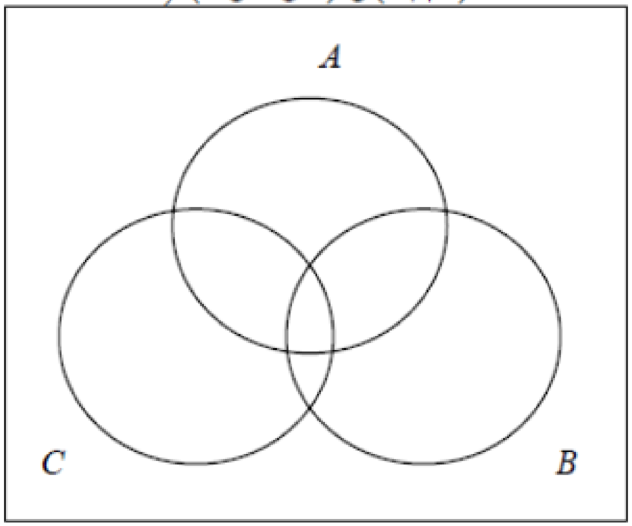


Example 11 Cont.: Let *A, B,* and *C* be subsets of *U*, use the Venn diagram to shade the solution:

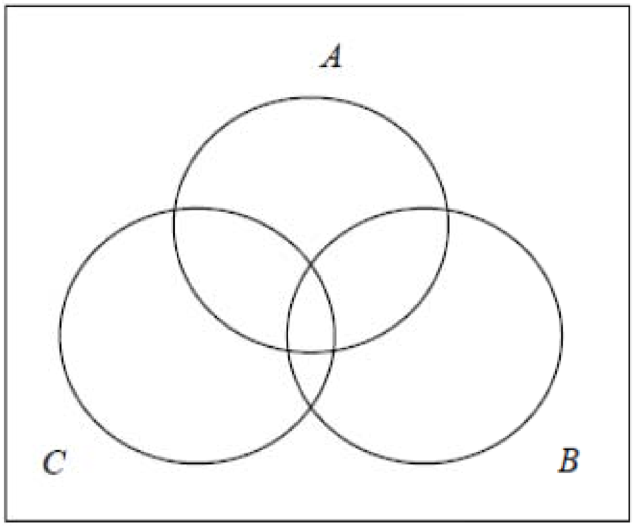
i) $(A \cap B') \cup C$



J) $(A \cup B \cup C)' \cup (A \cap B)$

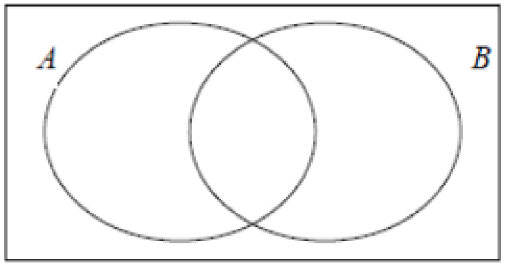


k) $(A \cup B \cup C)' \cap (A \cap B)$



Example 12: Which of the following statements is **<u>True</u>**?

a) $A' \cup B' = (A \cup B)'$



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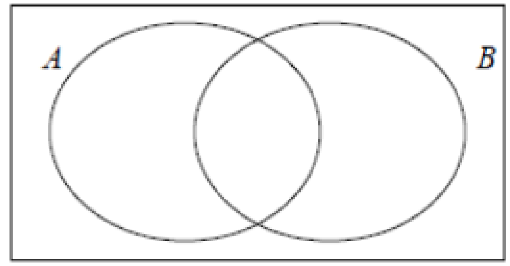
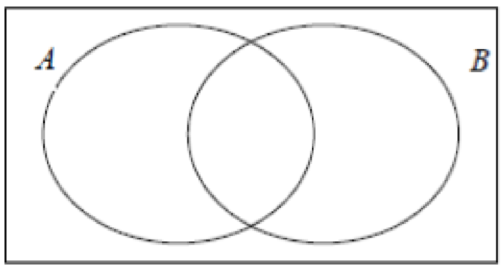


b) $A' \cap B' = (A \cap B)'$



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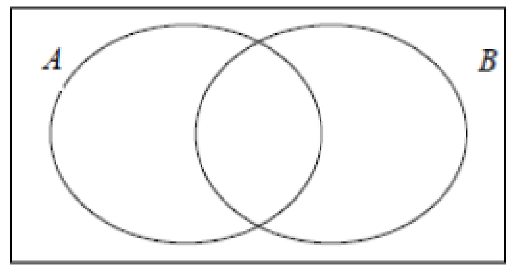
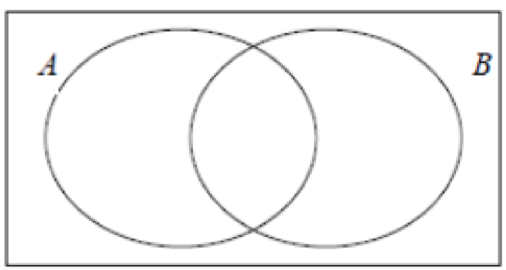


c) $A \cap B' \subseteq A' \cap B'$



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