Extra Handout on Finding the Inverse Matrix A^{-1}

To find the inverse matrix of $A = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$ using the All Integer Method:

- Step 1: Re-write it with the Identity Matrix I next to it on the right side:
  (The Identity Matrix I: the square matrix where all Diagonal elements = 1, the rest are zeros)

$$\left| \begin{array}{cc|cc} 2^* & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array} \right|$$

- Step 2: Do the pivot steps (2 pivots for two rows), and the last step should be:

$$\left| \begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \end{array} \right|$$

- Step 3: The Identity Matrix is now on the left side, and the Inverse Matrix in on the right side:

$$A^{-1} = \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix}$$

- You can check your answer by multiplying the original matrix A and the inverse A^{-1}. The answer must be an Identity Matrix I.

Note: Not every matrix has an inverse, for example: $A = \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix}$ does not have an inverse (the second pivot is zero).

Examples: Find the inverse matrix for each of the following and check your answer by multiplying the original matrix by its inverse, the resulting matrix must be an Identity Matrix:

1) $\begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix}$ 2) $\begin{vmatrix} 1 & 0 \\ 3 & -2 \end{vmatrix}$ 3) $\begin{vmatrix} 4 & -1 \\ 3 & -1 \end{vmatrix}$ 4) $\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$

5) $\begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix}$ 6) $\begin{vmatrix} 3 & 2 \\ -1 & 1 \end{vmatrix}$ 7) $\begin{vmatrix} -2 & -1 \\ 1 & 1 \end{vmatrix}$ 8) $\begin{vmatrix} 5 & 3 \\ 2 & 2 \end{vmatrix}$

9) $\begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix}$ 10) $\begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix}$ 11) $\begin{vmatrix} 5 & 3 \\ 3 & 2 \end{vmatrix}$ 12) $\begin{vmatrix} 3 & 5 \\ 1 & -2 \end{vmatrix}$

ANSWERS:

1) $\begin{vmatrix} 2 & -1/2 \\ -1 & 1/2 \end{vmatrix}$ 2) $\begin{vmatrix} 1 & 0 \\ 3/2 & -1/2 \end{vmatrix}$ 3) $\begin{vmatrix} 1 & -1 \\ 3 & -4 \end{vmatrix}$ 4) $\begin{vmatrix} 2 & -1 \\ -3 & 2 \end{vmatrix}$

5) $\begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix}$ 6) $\begin{vmatrix} 1/5 & -2/5 \\ 1/5 & 3/5 \end{vmatrix}$ 7) $\begin{vmatrix} -1 & -1 \\ 1 & 2 \end{vmatrix}$ 8) $\begin{vmatrix} 1/2 & -3/4 \\ -1/2 & 5/4 \end{vmatrix}$

9) N.P. 10) $\begin{vmatrix} 1 & -1/2 \\ -2 & 3/2 \end{vmatrix}$ 11) $\begin{vmatrix} 2 & -3 \\ -3 & 5 \end{vmatrix}$ 12) $\begin{vmatrix} 2/11 & 5/11 \\ 1/11 & -3/11 \end{vmatrix}$

Work and steps are shown on the following page if needed.

The following are the solutions and the steps of the previous examples. Use only if you did not get the correct answers given in the previous page:

1)

1*	1	1	0
2	4	0	1
1	1	1	0
0	2*	-2	1
2	0	4	-1
0	2	-2	1
1	0	**2**	**- 1/2**
0	1	**-1**	**1/2**

2)

1*	0	1	0
3	-2	0	1
1	0	1	0
0	-2*	-3	1
-2	0	-2	0
0	-2	-3	1
1	0	**1**	**0**
0	1	**3/2**	**- 1/2**

3)

4*	-1	1	0
3	-1	0	1
4	-1	1	0
0	-1*	-3	4
-1	0	-1	1
0	-1	-3	4
1	0	**1**	**-1**
0	1	**3**	**-4**

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4)

2*	1	1	0
3	2	0	1
2	1	1	0
0	1*	-3	2
1	0	**2**	**-1**
0	1	**-3**	**2**

5)

2*	1	1	0
-1	-1	0	1
2	1	1	0
0	-1*	1	2
-1	0	-1	-1
0	-1	1	2
1	0	**1**	**1**
0	1	**-1**	**-2**

6)

3*	2	1	0
-1	1	0	1
3	2	1	0
0	5*	1	3
5	0	1	-2
0	5	1	3
1	0	**1/5**	**- 2/5**
0	1	**1/5**	**3/5**

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7)

-2*	-1	1	0
1	1	0	1
-2	-1	1	0
0	-1*	-1	-2
-1	0	1	1
0	-1	-1	-2
1	0	**-1**	**-1**
0	1	**1**	**2**

8)

5*	3	1	0
2	2	0	1
5	3	1	0
0	4*	-2	5
4	0	2	-3
0	4	-2	5
1	0	**1/2**	**- 3/4**
0	1	**-1/2**	**5/4**

9)

2*	1	1	0
4	2	0	1
2	1	1	0
0	0*	-4	2

Not Possible, Zero Pivot

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10)

3*	1	1	0
4	2	0	1
3	1	1	0
0	2*	-4	3
2	0	2	-1
0	2	-4	3
1	0	**1**	**- 1/2**
0	1	**-2**	**3/2**

11)

5*	3	1	0
3	2	0	1
5	3	1	0
0	1*	-3	5
1	0	**2**	**-3**
0	1	**-3**	**5**

12)

3*	5	1	0
1	-2	0	1
3	5	1	0
0	-11*	-1	3
-11	0	-2	-5
0	-11	-1	3
1	0	**2/11**	**5/11**
0	1	**1/11**	**-3/11**

You should check your answer by multiplying the original matrix with its inverse, the resulting matrix must be an Identity Matrix or $I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$