DISCRETE LOGISTIC MAP

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In one-dimensional dynamics computer experiments are very useful. They are easy to perform and often provide an important insight into the dynamics of the map that one investigates. However, a computer operates on a finite set, so we would like to know how well the real system is approximated by what the computer is doing. Such questions have been addressed a long ago (for instance, P. Góra and A. Boyarsky, *Why computers like Lebesgue measure*, Comput. Math. Appl. **16** (1988), 321–329). However, while general questions are being asked, apparently nobody concentrates on the dynamics of concrete maps of finite sets that arise in this context. I propose one such model for investigation.

Let us look at the discrete versions of the logistic map $f(x) = 4x(1-x)$ on the interval $[0,1]$. For a given positive integer n we define a map $g_n : E_n \to E_n$, where $E_n = \{k/n : k = 0, 1, 2, \dots n\}$ as follows. Let $\varphi_n : [0,1] \to E_n$ map any point to its nearest neighbor from E_n, that is $\varphi_n(x) = k/n$ if $k - 0.5 \le nx < k + 0.5$. In other words, φ_n is a round-off map. Then we define $g_n = \varphi_n \circ f$.

Since E_n is a finite set, all orbits of g_n are periodic or eventually periodic. Many points of E_n close to 0.5 are mapped to 1, which in turn is mapped to 0. The preimages of those points are mapped to 0 in three steps, etc. Therefore one can predict that considerable number of points of E_n will be sooner or later mapped to 0. Let a_n be the number of those elements of E_n whose orbits never get to 0 under the iterates of g_n.

Computer experiments show that the sequence a_n behaves in an unpredictable manner. For example, $a_{34572} = 23480$, $a_{34573} = 26294$, $a_{34574} = 1064$, $a_{34575} = 0$, $a_{34576} = 27090$. In fact, those experiments show that from time to time we get $a_n = 0$. The largest n for which I know that this is the case, is $n = 125815$ (I did not try much larger numbers because of the computer time necessary for this). The number of iterates necessary for all orbits to fall into 0 is in this case 548. This is slightly larger than $\sqrt{2n}$, which makes sense heuristically.

Thus, we can state the following conjecture.

Conjecture: *For infinitely many integers n we have $a_n = 0$.*

In any case, analysis of the behavior of the sequence $(a_n)_{n=1}^{\infty}$ can be interesting. While I believe that some kind of statistical analysis may be possible, proving any dependence of a_n on number-theoretic properties of n may be extremely difficult.