Dense periodic points in cellular automata

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Devaney defines a topological dynamical system to be chaotic if it is sensitive to initial conditions, transitive, and has a dense set of periodic points; several authors afterwards proved that the two last conditions imply the first.

Cellular automata (CA) are all continuous shift–commuting self–maps acting on a full shift, or sometimes on a subshift of finite type. It is easy to prove that a transitive CA acting on the full shift depends sensitively on initial conditions. It was shown by Boyle and Kitchens [BK] that left–closing CA and right–closing CA have a dense set of periodic points; the same result was obtained by Blanchard and Tisseur [BT] for surjective non-sensitive CA. These two classes have a very small intersection. It is easy to prove that any CA has a dense set of *ultimately periodic* points.

The questions whether all surjective CA, or at least all transitive CA, have a dense set of properly *periodic points*, is still open. The answer, positive or negative, is a necessary step before one understands the meaning of chaos in this field.

References

[BK] M. Boyle, B. Kitchens, *Periodic points in cellular automata*, preprint (1999). To appear, Indagationes Math.

[BT] F. Blanchard, P. Tisseur, *Some properties of cellular automata with equicontinuity points.* (avec P. Tisseur). Ann. Inst. Henri Poincaré, Probabilités et Statistiques **36** (2000) 569-582.