Honors Project 15: The Operation of a Diesel Engine

Objective

In this project we discuss computation of the work done by a diesel engine and the efficiency of such an engine.

Background Required

The Fundamental Theorem of Calculus.

Narrative

A typical diesel engine cylinder is illustrated in Fig. 1. Observe that diesel engines have no spark plugs. Diesel engines work on the principle that when a gas such as air is compressed, its temperature rises.

Diesel engines are two-cycle engines: a diesel cylinder has a compression stroke and a power stroke. Air is compressed during the compression stroke (see Fig. 2). By the time the piston has reached "top dead center", the temperature inside the cylinder will have reached 800°–1000° Fahrenheit, well above the ignition temperature of the fuel. Just after the piston has reached “top dead center”, fuel is injected into the cylinder. It ignites and drives the piston down through its power cycle. When the head nears the bottom of the power cycle, clean air enters through ports on the side of the cylinder at the same time an exhaust valve opens at the top of the cylinder to exhaust the spent air/fuel mixture.

If we plot pressure against volume for a diesel engine cylinder, we obtain a figure as illustrated in Fig. 2. During the compression stroke, the piston moves up and pressure inside the cylinder increases as volume decreases. At (\(V_1, P_1\)) fuel enters the cylinder. Observe that after fuel has entered the cylinder but before the power stroke, the pressure drops and the volume increases slightly (since the input valve is open). (We have exaggerated this part of Fig. 2 for the purposes of presentation.) The fuel ignites at (\(V_2, P_2\)) sending the piston down through the power stroke. At (\(V_3, P_3\)) the piston is at the bottom of the power stroke, and pressure inside the cylinder drops as air enters the cylinder and the spent air/fuel mixture is exhausted. At (\(V_4, P_4\)) the cylinder enters its next compression cycle.
To simplify computations in this project we assume (see Fig. 3) that no pressure is lost when fuel enters the cylinder, and that the exchange of spent air/fuel mixture for clean air is instantaneous (at constant volume). Thus, we assume $P_1 = P_2$, and that $V_3 = V_4$. Further, we assume that conditions are such that pressure and volume are related along the compression and power curves by the equations

$$PV^\gamma = k_c \quad \text{and} \quad PV^\gamma = k_p$$

for some constants $\gamma > 1$, $k_c$ and $k_p$.$^1$

**Tasks**

1. Prove that:
   (a) along the compression curve $P = P_1 V_1^\gamma / V_1^\gamma$,
   (b) along the power curve $P = P_1 V_2^\gamma / V_2^\gamma$,
   (c) $\left(\frac{V_1}{V_4}\right)^\gamma = \frac{P_4}{P_1}$ and $\left(\frac{V_2}{V_3}\right)^\gamma = \frac{P_3}{P_2}$

2. Assuming that $l = 0$ is the position of the head of the piston at $(V_4, P_4)$ and that $l = L$ is the position of the head of the piston at $(V_1, P_1)$,
   (a) explain why the work $W_c$ done by the cylinder during the compression stroke from $(V_4, P_4)$ to $(V_1, P_1)$ is
   $$W_c = - \int_{l=0}^{L} F \, dl = - \int_{l=0}^{L} PA \, dl = - \int_{V=V_4}^{V_1} P \, dV$$

   where $A$ is the area of the piston head,
   (b) show that
   $$W_c = \frac{1}{\gamma - 1} (P_1 V_1 - P_4 V_4)$$
   (c) explain why the work $W_p$ done by the cylinder during the power stroke from $(V_1, P_1)$ to $(V_3, P_3)$ is
   $$W_p = P_1 (V_2 - V_1) + \int_{V=V_2}^{V_3} P \, dV$$

$^1$Justifying the pressure-volume relationship along the compression and power curves requires a discussion of thermodynamics beyond the scope of this project. The interested reader should see [1] for such a discussion.
(d) show that

\[ W_p = \frac{1}{\gamma - 1} (\gamma P_1 (V_2 - V_1) + P_1 V_1 - P_3 V_3) \]

(e) find a formula for the net work \( W_p - W_c \) done by the cylinder in one complete compression/power cycle,

(f) find a formula for the efficiency

\[ E = \frac{W_p - W_c}{W_p} = 1 - \frac{W_c}{W_p} \]

(g) assuming \( T_1, T_2, T_3, T_4 \) are the temperatures of the gases in the cylinder at \( (V_1, P_1) \), \( (V_2, P_2) \), \( (V_3, P_3) \), \( (V_4, P_4) \), and the Gas Law \( PV = nRT \), so that

\[ \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} = \frac{P_4 V_4}{T_4} \]

find a formula for the efficiency \( E \) solely in terms of \( \gamma \) and the temperatures \( T_1, T_2, T_3, T_4 \).

The formulas derived in this project are important in the theory and design of diesel engines.

References
