Speaker: Ziting Tang  
Department of Mathematical Sciences, IUPUI

Title: Asymptotic Behavior of M-Estimators of p Regression Parameters when $p^2/n$ is Large. I. Consistency

Abstract:
Consider the general linear model $Y = x\beta + R$ with $Y$ and $R$ $n$-dimensional, $\beta$ $p$-dimensional, and $x$ an $n \times p$ matrix with rows $x_i'$. Let $\psi$ be given and let $\hat{\beta}$ be an M-estimator of $\beta$ satisfying $0 = \sum x_i \psi (Y_i - x_i' \hat{\beta})$. Previous authors have considered consistency and asymptotic normality of $\hat{\beta}$ when $p$ is permitted to grow, but they have required at least $p^2/n \to 0$. Here the following result is presented: in typical regression cases, under reasonable conditions if $p (\log p)/n \to 0$ then $\|\hat{\beta} - \beta\|_2 = O_p(p/n)$. A subsequent paper will show that $\hat{\beta}$ has a normal approximation in $\mathbb{R}^p$ if $(p \log p)^{3/2}/n \to 0$ and that $\max_i |x_i' (\hat{\beta} - \beta)| \overset{p}{\to} 0$ (which would not follow from norm consistency if $p^2/n \to \infty$). In ANOVA cases, $\hat{\beta}$ is not norm consistent, but it is shown here that $\max_i |x_i' (\hat{\beta} - \beta)| \overset{p}{\to} 0$ if $p \log p/n \to 0$. A normality result for arbitrary linear combinations $a' (\hat{\beta} - \beta)$ is also presented in this case.