Indiana University-Purdue University Indianapolis
Department of Mathematical Sciences

Statistics Seminar

12:15pm—1:15pm, Tuesday, February 19, 2019
LD 265

Speaker: Ziting Tang
Department of Mathematical Sciences, IUPUI

Title: Asymptotic Behavior of M-Estimators of p Regression Parameters when p^2/n is Large. I. Consistency

Abstract:

Consider the general linear model $Y = x\beta + R$ with Y and R n-dimensional, β p-dimensional, and x an $n \times p$ matrix with rows x_i'. Let ψ be given and let $\hat{\beta}$ be an M-estimator of β satisfying $0 = \sum x_i \psi(Y_i - x_i'\hat{\beta})$. Previous authors have considered consistency and asymptotic normality of $\hat{\beta}$ when p is permitted to grow, but they have required at least $p^2/n \to 0$. Here the following result is presented: in typical regression cases, under reasonable conditions if $p(\log p)/n \to 0$ then $\|\hat{\beta} - \beta\|^2 = \mathcal{O}_p(p/n)$. A subsequent paper will show that $\hat{\beta}$ has a normal approximation in R^p if $(p \log p)^{3/2}/n \to 0$ and that $\max_i |x_i'(\hat{\beta} - \beta)| \xrightarrow{p} 0$ (which would not follow from norm consistency if $p^2/n \to \infty$). In ANOVA cases, $\hat{\beta}$ is not norm consistent, but it is shown here that $\max_i |x_i'(\hat{\beta} - \beta)| \xrightarrow{p} 0$ if $p \log p/n \to 0$. A normality result for arbitrary linear combinations $a'(\hat{\beta} - \beta)$ is also presented in this case.