Title: Gröbner bases for vanishing ideals and multivariate interpolation

Abstract:

Gröbner basis is a particular kind of generating set of a polynomial ideal that have been applied in symbolic computations, coding theory, statistics and numerical analysis etc. A vanishing ideal $I(P)$ is the set of polynomials that vanish at given finite point set $P$. Finding Gröbner bases and associated Gröbner escaliers of $I(P)$ is one of the most concerned problems of computational algebraic geometry. For example, if a design $D$ is seen as a finite point set in a finite dimensional space, then the Gröbner éscalier of $I(D)$ is the support of an estimable model with respect to $D$.

The structures of Gröbner bases and Gröbner éscaliers of $I(P)$ depend significantly on the geometry of $P$, but traditional algorithms do not take this into account. We investigate Gröbner bases and Gröbner éscaliers of the vanishing ideals of three types of point sets with special geometries, and then with their help we improved the efficiency of traditional algorithms.

Gröbner éscaliers of $I(P)$ can also be applied in the field of polynomial interpolation and rational interpolation. If we choose a degree compatible monomial order, minimal degree Lagrange interpolation bases and Newton interpolation bases, an extension of classical univariate interpolation, can be obtained from the space spanned by the Gröbner éscalier. Further rational interpolation can also be solved partially.