Overview

Model Assumptions

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Computation and comparison

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Conclusions and Future Work

References
A two-stage shock model with self-healing mechanism is proposed: Change point is defined as the moment when the cumulative number of valid shocks reaches $d$.

Before the change point, the system can heal when $\delta$-invalid shocks reach $k$ in the trailing run of invalid shocks. Equivalently, the damage caused by previous $i$ valid shocks can be healed when the number of delta-invalid shocks falls in $[ik, (i+1)k)$ in the run of invalid shocks.
The system loses self-healing ability upon reaching change point, and then fails when the cumulative number of valid shocks reaches a prefixed value $n(n > d)$.

Finite Markov chain imbedding approach is employed to obtain the pmf, cdf and the mean of shock length.

Three preventive maintenance policies are proposed for the system under different monitoring condition.
Valid shock: a shock that results in a certain degree of system damage.

\( \delta \)- invalid shock: an invalid shock whose time lag with the preceding shock exceeds a given threshold \( \delta \).
The system is operating in random environment, which is subject to a sequence of randomly occurring shocks over time.

Each shock has two possible types: Valid shock with the probability of $p$, and invalid shock with the probability of $q$ (i.e., $p + q = 1$).

$X_i$ is time lag between the $(i - 1)$-th and $i$-th shock, $i \geq 1$. $X_i$ are i.i.d $\sim Exp(\lambda)$. They are independent of type of shock.
Consider the example of aviation industry. An aircraft can be considered as a functioning system exposed to shocks.

- **Valid Shocks**: Turbulences that can cause damage to the airframe.
- **δ-invalid shocks**: External stimuli like heat, light, electrical fields or moisture.
- These stimuli can trigger self-healing mechanism that heals the system from damages.
- When the cumulative damage reaches a certain amount, the aircraft will lose the self-healing ability and then the damage caused by valid shocks starts to accumulate.
- The reliability of aircraft can be increased and its service life can be extended effectively by applying the self-healing materials.
Failure process of the System

- If a valid shock occurs, the cumulative number of valid shocks should be added by 1.
Failure process of the System

- If a valid shock occurs, the cumulative number of valid shocks should be added by 1.
- If the number of $\delta$-invalid shocks reaches $k$ in the trailing run of invalid shocks, the cumulative number of valid shocks should be reduced by 1.

Let us consider $n = 5$, $d = 3$, $k = 2$, $\delta = 1$. Consider $1, 0', 0$, $I$, $(X_1, \ldots, X_{11})$ indicate valid shocks, invalid shocks, the initial state of the sequence of shocks and the time lags, respectively.
If a valid shock occurs, the cumulative number of valid shocks should be added by 1.

If the number of $\delta$-invalid shocks reaches $k$ in the trailing run of invalid shocks, the cumulative number of valid shocks should be reduced by 1.

Next self-healing behavior is triggered if the number of additional $\delta$-invalid shocks reaches $k$ among this run of invalid shocks. The cumulative number of valid shocks remain unchanged if it becomes 0. Otherwise, it should be reduced by 1 again.
Failure process of the System

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- If the number of $\delta$-invalid shocks reaches $k$ in the trailing run of invalid shocks, the cumulative number of valid shocks should be reduced by 1.
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Let $n$ : cumulative number of valid shocks that cause the system failure, $d$ : cumulative number of valid shocks that make the system come to the change point.
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$n$ : cumulative number of valid shocks that cause the system failure
$d$ : cumulative number of valid shocks that make the system come to the change point

Let us consider $n = 5$, $d = 3$, $k = 2$, $\delta = 1$. Consider $1'$, $0'$, $0_1$, $(X_1, \ldots, X_{11})$ indicate valid shocks, invalid shocks, the initial state of the sequence of shocks and the time lags, respectively.
For $n = 6$, $d = 3$, $k = 2$, $\delta = 1$
The Picturization of the two-stage model

Therefore essentially, the two-stage process looks like this:

![Diagram of two-stage model]

**Fig. 1.** Two-stage failure process of the system.
Reliability Analysis of the shock model

- $S_m$ denotes the cumulative number of valid shocks.
- $T_m$ represents the number of $\delta$-invalid shocks in the trailing run of invalid shocks.
- $R_m$ denotes the number of $\delta$-invalid shocks which can trigger a next self-healing behavior if the system is in stage 1 where 
  \[ R_m = T_m - \left\lfloor \frac{T_m}{k} \right\rfloor k, \] 
  otherwise, $R_m = \Theta$ where $\Theta$ means the system is in stage 2.
- Markov chain $Y_m, m \geq 0$ associated with $S_m$ and $R_m$ is defined as 
  \[ Y_m = (S_m, R_m), m \geq 0 \]
The state-space

\[ \Omega_m = \{(s, r) : 0 \leq s \leq d-1, 0 \leq r \leq k-1\} \cup \{(s, \Theta) : d \leq s \leq n-1\} \cup \{E_a\} \]

Initial State: \( Y_0 = (0, 0) \)

\( E_a \): absorbing state, cumulative number of valid shocks reaches \( n \) and the system fails.
The probability $q$ of invalid shock can be further broken down:

- $q_1 =$ probability that m-th shock is $\delta$-invalid.
- $q_2 =$ probability that m-th shock is an invalid shock but not $\delta$-invalid. 
  
  ($q_1 + q_2 = q$)

Example: $n = 4$, $d = 2$, $k = 2$

$$\Omega_m = \{(0, 0), (0, 1), (1, 0), (1, 1), (2, \Theta), (3, \Theta)\} \cup \{E_a\}$$
Fig. 3. State transition diagram for the Markov chain.
One Step transition probability Matrix

The one step transition probability matrix for the state-space is as follows:

\[
\begin{bmatrix}
(0,0) & q_2 & q_1 & p & 0 & 0 & 0 & 0 & 0 \\
(0,1) & q_1 & q_2 & p & 0 & 0 & 0 & 0 & 0 \\
(1,0) & 0 & 0 & q_2 & q_1 & p & 0 & 0 & 0 \\
(1,1) & q_1 & 0 & 0 & q_2 & p & 0 & 0 & 0 \\
(2,\Theta) & 0 & 0 & 0 & 0 & q & p & 0 & 0 \\
(3,\Theta) & 0 & 0 & 0 & 0 & 0 & q & p & 0 \\
E_a & 0 & 0 & 0 & 0 & 0 & 0 & q & p \frac{1}{7 	imes 7}
\end{bmatrix}
\]
Therefore the matrix $\Lambda$

\[
\Lambda = \begin{bmatrix}
(0,0) & [q_2 & q_1 & p & 0 & 0 & 0 & 0] \\
(0,1) & [q_1 & q_2 & p & 0 & 0 & 0 & 0] \\
(1,0) & [0 & 0 & q_2 & q_1 & p & 0 & 0] \\
(1,1) & [q_1 & 0 & 0 & q_2 & p & 0 & 0] \\
(2,\Theta) & [0 & 0 & 0 & 0 & q & p & 0] \\
(3,\Theta) & [0 & 0 & 0 & 0 & 0 & q & p] \\
E_a & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\end{bmatrix}
\]

Can be written in general in the following format:

\[
\Lambda = \begin{bmatrix}
A_m & B_m & \cdots & B_m \\
D_m & C_m & \cdots & D_m \\
& & C_m & E_m \\
& & & F_m & G_m \\
& & & & & & 1
\end{bmatrix}
\]

Where $\gamma = [dk + (n - d + 1)]$ is the cardinality of the state-space $\Omega$. 
Λ can be transitioned as follows: 

\[ \Lambda = \begin{bmatrix} P^{(\gamma-1) \times (\gamma-1)} & Q^{(\gamma-1) \times 1} \\ 0^{1 \times (\gamma-1)} & E^{1 \times 1} \end{bmatrix} \]

One step transition probabilities:
- \( P^{(\gamma-1) \times (\gamma-1)} \) among transient states
- \( Q^{(\gamma-1) \times 1} \) transient states to absorbing states
- \( 0^{1 \times (\gamma-1)} \) absorbing state to transient state
- \( E^{1 \times 1} \) identity matrix
Λ can be transitioned as follows: $\Lambda = \begin{bmatrix} P_{(\gamma-1)X(\gamma-1)} & Q_{(\gamma-1)X1} \\ 0_{1X(\gamma-1)} & E_{1X1} \end{bmatrix}$

One step transition probabilities:
- $P_{(\gamma-1)X(\gamma-1)}$ among transient states
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- $E_{1X1}$ identity matrix

Combining transient states in **stage 2** and absorbing states into new absorbing state $E_f$ (system moves from **stage 1**). $\Phi = \begin{bmatrix} U_{(\gamma_1)X(\gamma_1)} & V_{(\gamma_1)X1} \\ 0_{1X(\gamma_1)} & E_{1X1} \end{bmatrix}$ where $\gamma_1 = dXk$
The general format of matrices $P$, $Q$, $U$ and $V$
The distribution of $N_1$

$N_1$ is the total number of shocks until change point appears. Let $\alpha = (1, 0, \ldots, 0)_{1 \times \gamma_1}$, $I_c = U^0$ and $e_{\gamma_1} = (1, \ldots, 1)_{1 \times \gamma_1}$

The pmf of the shock length $N_1$ is

$$P(N_1 = k) = \alpha U^{k-1} V$$

The cdf of the shock length $N_1$ is

$$P(N_1 \leq k) = \sum_{i=1}^{k} \alpha U^{i-1} V$$

The expected shock length is

$$E(N_1) = \alpha (I_c - U)^{-1} e_{\gamma_1}$$
The distribution of $N_2$

$N_2$ is the total number of shocks until the system fails. Let $\pi_0 = (1, 0, \ldots, 0)_{\mathbb{1}X(\gamma-1)}$, $I = P^0$ and $e_{(\gamma-1)} = (1, \ldots, 1)_{\mathbb{1}X(\gamma-1)}$. The pmf of the shock length $N_2$ is

$$P(N_2 = k) = \pi_0 P^{k-1} Q$$

The cdf of the shock length $N_2$ is

$$P(N_2 \leq k) = \sum_{i=1}^{k} \pi_0 P^{i-1} Q$$

The expected shock length is

$$E(N_2) = \pi_0 (I - P)^{-1} e_{\gamma-1}$$
Consider time lags between two successive shocks follow the exponential distribution with parameter $\lambda$.

$n = 6, d = 3, k = 2$ the state space is $\Omega_m = \{(0, 0), (0, 1), (1, 0), (1, 1), (2, 0), (2, 1), (3, \Theta), (4, \Theta), (5, \Theta), \} \cup \{E_a\}$.

We consider values of parameters are chosen to be $\lambda = 0.5, \delta = 1.5, p = 0.4$. Then, $q_1 = q_P\{X_i > \delta\} = 0.2834$, $q_2 = q_P\{X_i \leq \delta\} = 0.3166$. The transition probability matrices are shown in next slide.

After calculation, we find $E(N_1) = 8.6464$, $E(N_2) = 16.1464$. 
Illustrative Example (Contd.)

\[
\Lambda =
\begin{bmatrix}
(0,0) & q_2 & q_1 & p & 0 & 0 & 0 & 0 & 0 & 0 \\
(0,1) & q_1 & q_2 & p & 0 & 0 & 0 & 0 & 0 & 0 \\
(1,0) & 0 & 0 & q_2 & q_1 & p & 0 & 0 & 0 & 0 \\
(1,1) & q_1 & 0 & 0 & q_2 & p & 0 & 0 & 0 & 0 \\
(2,0) & 0 & 0 & 0 & 0 & q_2 & q_1 & p & 0 & 0 \\
(2,1) & 0 & 0 & q_1 & 0 & 0 & q_2 & p & 0 & 0 \\
(3,\Theta) & 0 & 0 & 0 & 0 & 0 & 0 & q & p & 0 & 0 \\
(4,\Theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q & p & 0 \\
(5,\Theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q & p \\
E_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 10x10
\end{bmatrix}
\]
Illustrative Example (Contd.)

\[
P = \begin{bmatrix}
q_2 & q_1 & p & 0 & 0 & 0 & 0 & 0 & 0 \\
q_1 & q_2 & p & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & q_2 & q_1 & p & 0 & 0 & 0 & 0 \\
q_1 & 0 & 0 & q_2 & p & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & q_2 & q_1 & p & 0 & 0 \\
0 & 0 & q_1 & 0 & 0 & 0 & q_2 & p & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & q & p \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\quad Q = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}_{9 \times 9}
\]

\[
U = \begin{bmatrix}
q_2 & q_1 & p & 0 & 0 & 0 \\
q_1 & q_2 & p & 0 & 0 & 0 \\
0 & 0 & q_2 & q_1 & p & 0 \\
q_1 & 0 & 0 & q_2 & p & 0 \\
0 & 0 & 0 & 0 & q_2 & q_1 \\
0 & 0 & q_1 & 0 & 0 & 0
\end{bmatrix}_{6 \times 6},
\quad V = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}_{6 \times 1}
\]
## Table for some other choices of parameters

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Thank you!