IUPUI Department of Mathematical Sciences
Departmental Final Examination

PRACTICE FINAL EXAM VERSION #1

MATH 15400

Trigonometry

Exam directions similar to those on the departmental final.

1. DO NOT OPEN this test booklet until you are told to do so.
2. This is NOT the exam for MATH 15300 or 15900.
3. There are 8 pages in this exam with problems 1 to 24 and a bonus problem.
4. You MUST get a new exam from the proctor if your exam is incomplete.
5. PRINT your name and student ID# below.
6. MARK your section below.
7. You will have two hours to complete this examination.
8. A TI-30Xa calculator is permitted, no other calculator is allowed.
9. No scrap paper, notes, books, or collaborators are allowed.
10. Exact answers may contain π or radicals or logarithms.
11. Simplify all answers completely.
12. Problems involving units must have the units represented on the answer to receive full credit.

Name (Print Clearly)	Solutions
Student ID#	

Practice Departmental Final Exam Recommendations to Students:

- Take this practice final exam like an actual examination (not like doing homework). That is, create an "exam like" atmosphere. This practice exam should be taken after completing a thorough review of the material.
- Set aside a two-hour block of time with no interruptions (no facebook, texting, phone calls, restroom breaks, etc.).
- Do not use any help aids, such as notes, textbook, internet, scrap paper, MAC staff, etc.
- Work through all problems noting which concepts you know well and which ones you need to spend more time on.
- Grade your exam using the answers in the back of your textbook (the textbook section and exercise number is noted at the top right of each problem).
- Rework any problem on the exam that you missed and then work similar problems from the textbook until you can perform the operations without error.
- Follow the same recommendations for taking the Practice Final Exam Version #2.

MATH 15400 Practice Departmental Final Exam (Version #1)

TEXTBOOK: Swokowski & Cole, *Algebra & Trigonometry with Analytic Geometry*, Classic 12th Edition

To receive full credit you must show all your work. Simplify all answers completely. Be sure to check your final answers for errors. Problems involving units must have the units represented on the answer to receive full credit.

1. Find the vertex, focus, and directrix of the parabola. (11.1 #9)

$y = x^2 - 4x + 2$ Vertical $(x-h)^2 = 4p(y-k)$

$x^2 - 4x = y - 2$

$x^2 - 4x + 4 = y - 2 + 4$ $F(h, k+p)$ $F(2, -2 + 1/4)$

$(x-2)^2 = 1(y+2)$ $F(2, -7/4)$

$V(2,-2)$ $4p = 1$ $y = k - p$

$p = 1/4$ $y = -2 - 1/4$, $y = -9/4$

1. $V(2,-2)$, $F(2, -7/4)$, $y = -9/4$ (4)

2. Find an equation of the ellipse that has its center at the origin with horizontal major axis of length 8, minor axis of length 5. (11.2 #29)

Horizontal $2a = 8$ $2b = 5$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $a = 4$ $b = 5/2$

$a^2 = 16$ $b^2 = 25/4$

$\frac{x^2}{16} + \frac{y^2}{25/4} = 1$ $\frac{x^2}{16} + \frac{4y^2}{25} = 1$

2. $\frac{x^2}{16} + \frac{4y^2}{25} = 1$ (4)

3. Find the center, vertices, the foci, and the equations of the asymptotes of the hyperbola. Sketch its graph.

$\frac{(y+2)^2}{9} - \frac{(x+2)^2}{4} = 1$ VERTICAL (11.3 #11)

(4)

$C(-2,-2)$

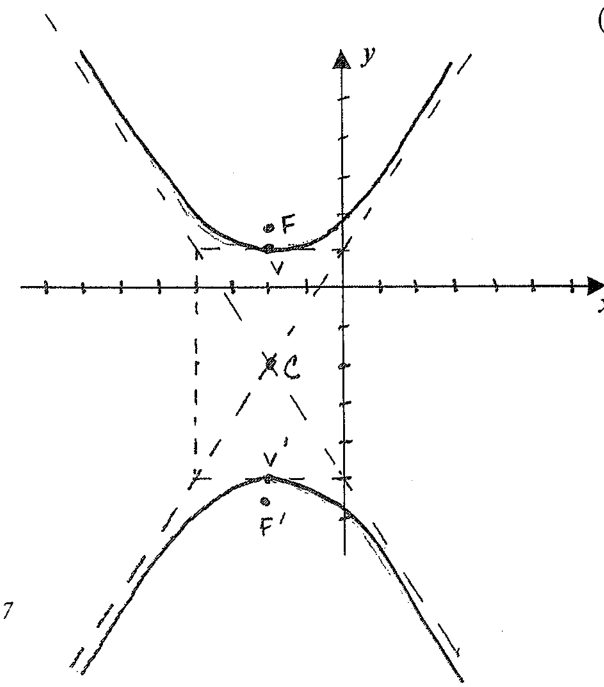
$V(-2, -2 \pm 3)$

$W(-2 \pm 2, -2)$

$c^2 = a^2 + b^2$, $c^2 = 13$, $c = \sqrt{13}$

$F(-2, -2 \pm \sqrt{13})$

$(y+2) = \pm 3/2 (x+2)$



4. Solve the system. (9.1 #25)

$$\begin{cases} y^2 - 4x^2 = 4 \\ 9y^2 + 16x^2 = 140 \end{cases}$$

$$\begin{cases} 4y^2 - 16x^2 = 16 \\ 9y^2 + 16x^2 = 140 \end{cases}$$

$13y^2 = 156$

$y^2 = 12$

$y = \pm\sqrt{12} = \pm 2\sqrt{3}$

$12 - 4x^2 = 4$

$-4x^2 = -8$

$x^2 = 2$

$x = \pm\sqrt{2}$

4 points: $(\pm\sqrt{2}, \pm 2\sqrt{3})$

4. $(\pm\sqrt{2}, \pm 2\sqrt{3})$ (4)

5. ***Mixing a silver alloy*** A silversmith has two alloys, one containing 35% silver and the other 60% silver. How much of each should be melted and combined to obtain 100 grams of an alloy containing 50% silver? (9.2 #31)

Let x = amount of 35% silver alloy

y = " " 60% " "

$$\begin{cases} 35x + 60y = 50(100) \\ x + y = 100 \end{cases}$$

$35x + 60y = 5000$

$-\ 35x - 35y = -3500$

$25y = 1500$

$y = 60g$

$x = 40g$

5. 40g of 35% silver alloy
60g of 60% silver alloy (4)

6. a) Find the radian and degree measures of the central angle θ subtended by an arc of 7 cm on a circle of radius 4 cm. (6.1 #33)

$s = r\theta$

$\theta = \frac{s}{r}$

$\theta = \frac{7\text{cm}}{4\text{cm}} = \frac{7}{4}$ radians

$\theta = \frac{7}{4}\left(\frac{180^\circ}{\pi}\right) = \left(\frac{315}{\pi}\right)^\circ \approx 100.27^\circ$

6a) $\theta = \frac{7}{4}$ radians
$\theta = \left(\frac{315}{\pi}\right)^\circ$ (4)

b) Find the area of the sector determined by θ in part (a).

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(4)^2\left(\frac{7}{4}\right) = 14\text{ cm}^2$

6b) $A = 14\text{ cm}^2$ (4)

7. A wheel of radius 5 inches is rotating at a rate of 40 rpm. (6.1 #45)

a) Find the angular speed (in radians per minute).

$$\omega = \frac{40\text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ radians}}{1\text{ rev}}$$

$$\omega = 80\pi \text{ rad/min}$$

7a) 80π rad/min (4)

b) Find the linear speed of a point on the circumference (in ft/min).

$$V = r\omega$$

$$V = \left(\frac{5\text{ in}}{1}\right)\left(\frac{80\pi}{\text{min}}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right) = \frac{100\pi}{3}\text{ ft/min}$$

7b) $\frac{100\pi}{3}$ ft/min (4)

8. Find the exact values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ for the acute angle θ if $\sec\theta = \frac{6}{5}$. (6.2 #21)

$\sec\theta = \frac{r}{x}$

Let $x = 5$

$y = \sqrt{11}$

$r = 6$

$r^2 = x^2 + y^2$

$36 = 25 + y^2$

$y^2 = 11$

$y = \pm\sqrt{11}$

$y = \sqrt{11}$ since θ is an acute angle

$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

8. $\sin\theta = \frac{\sqrt{11}}{6}$, $\tan\theta = \frac{\sqrt{11}}{5}$, $\cos\theta = \frac{5}{6}$ (4)

9. Find the **exact** value. (6.4 #18)

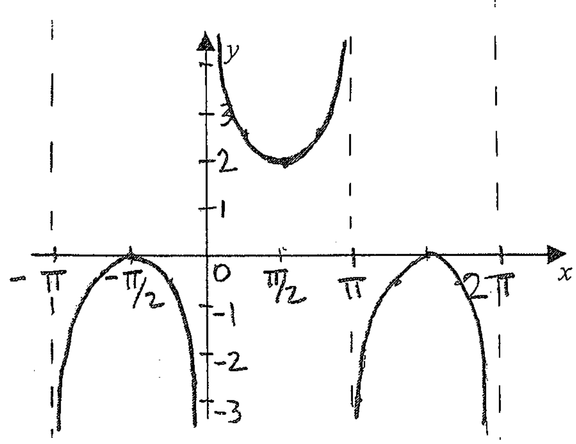
$$\csc(-2\pi/3) = \frac{1}{\sin(-2\pi/3)} = \frac{1}{-\sin 2\pi/3} = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}}$$

9. $-\frac{2}{\sqrt{3}}$ (4)

10. Graph at least one complete period of $y = 1 + \csc x$. (6.3 #62)

(4)

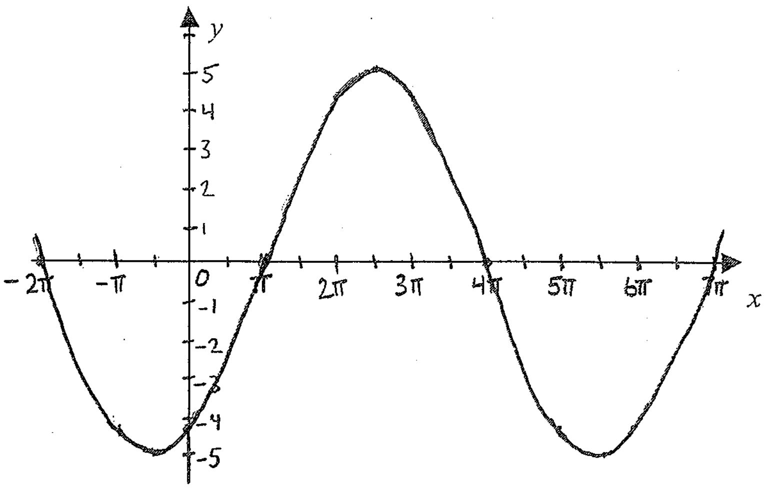
Shift the graph of $y = \csc x$ up 1 unit.



11. Find the amplitude, period, phase shift, and graph at least one complete period for $y = -5\cos\left(\frac{1}{3}x + \frac{\pi}{6}\right)$.

(6.5 #31)

(4)



$0 \le \frac{1}{3}x + \frac{\pi}{6} \le 2\pi$

$-\pi/6 \le \frac{1}{3}x \le \frac{11\pi}{6}$

$-\frac{\pi}{2} \le x \le \frac{11\pi}{2}$, phase shift $x = -\pi/2$

period $= 6\pi$

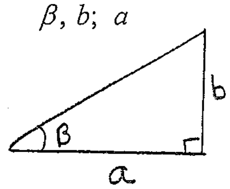
amplitude $= 5$

y-intercept $\left(0, -\frac{5\sqrt{3}}{2}\right)$

12. Given the indicated parts of triangle ABC with $\gamma = 90°$, express the third part in terms of the first two.

(6.7 #19)

β, b; a



$\tan\beta = \frac{b}{a}$

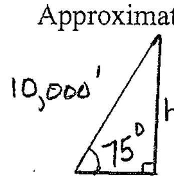
$a = \frac{b}{\tan\beta}$ or $a = b\cot\beta$

$a = \frac{b}{\tan\beta}$ or

12. $a = b\cot\beta$ (4)

13. A rocket is fired at sea level and climbs at a constant angle of 75° through a distance of 10,000 feet. Approximate its altitude to the nearest foot.

(6.7 #31)



$\sin 75° = \frac{h}{10000}$

$h = 10{,}000 \sin 75°$ feet

$h \approx 9659$ feet

13. $h \approx 9659$ ft (4)

14. Verify the identity. **Show all work.**

(7.1 #23)

(4)

$\tan^4 k - \sec^4 k = 1 - 2\sec^2 k$

$$\tan^4 K - \sec^4 K = (\tan^2 K - \sec^2 K)(\tan^2 K + \sec^2 K)$$
$$= (-1)(\sec^2 K - 1 + \sec^2 K)$$
$$= (-1)(2\sec^2 K - 1)$$
$$= 1 - 2\sec^2 K$$

15. Find the **exact values** of the solutions of the equation that are in the interval $[0,2\pi)$. (7.2 #43)

$2\sin^2 u = 1 - \sin u$

$2\sin^2 u + \sin u - 1 = 0$

$(2\sin u - 1)(\sin u + 1) = 0$

$\sin u = 1/2, \quad \sin u = -1$

$u = \pi/6, 5\pi/6, \quad u = 3\pi/2$

15. $u = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ (4)

16. If $\cos\alpha = -\frac{2}{5}$ and $\cos\beta = -\frac{3}{5}$ for third-quadrant angles α and β, find the **exact value** for: (7.3 #23)

α: QIII $x = -2$, $y = -\sqrt{21}$, $r = 5$

β: QIII $x = -3$, $y = -4$, $r = 5$

a) $\sin(\alpha - \beta)$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \left(-\frac{\sqrt{21}}{5}\right)\left(-\frac{3}{5}\right) - \left(-\frac{2}{5}\right)\left(-\frac{4}{5}\right) = \frac{3\sqrt{21} - 8}{25}$

16a) $\frac{3\sqrt{21} - 8}{25}$ (4)

b) $\cos(\alpha - \beta)$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \left(-\frac{2}{5}\right)\left(-\frac{3}{5}\right) + \left(-\frac{\sqrt{21}}{5}\right)\left(-\frac{4}{5}\right) = \frac{6 + 4\sqrt{21}}{25}$

16b) $\frac{4\sqrt{21} + 6}{25}$ (4)

17. Given $\sec\theta = -3;\ 90° < \theta < 180°$ find the **exact value** of $\sin 2\theta$, $\cos 2\theta$ and $\tan 2\theta$. (7.4 #3)

$\cos\theta = -\frac{1}{3}$ QII

$x = -1$

$y = \sqrt{8} = 2\sqrt{2}$

$r = 3$

$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{2\sqrt{2}}{3}\right)\left(-\frac{1}{3}\right) = -\frac{4\sqrt{2}}{9}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{1}{3}\right)^2 - \left(\frac{\sqrt{8}}{3}\right)^2 = -\frac{7}{9}$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2(-2\sqrt{2})}{1 - (-2\sqrt{2})^2}$

$= \frac{-4\sqrt{2}}{-7} = \frac{4\sqrt{2}}{7}$

17a) $\sin 2\theta$: $\frac{-4\sqrt{2}}{9}$ (4)

17b) $\cos 2\theta$: $-\frac{7}{9}$ (4)

17c) $\tan 2\theta$: $\frac{4\sqrt{2}}{7}$ (4)

18. Find the **exact values** of the solutions of the equation that are in the interval $[0, 2\pi)$. (7.4 #35)

$\sin 2t + \sin t = 0$

$2\sin t\cos t + \sin t = 0$

$\sin t\,(2\cos t + 1) = 0$

$\sin t = 0 \quad \cos t = -1/2$

$t = 0, \pi \quad t = 2\pi/3, 4\pi/3$

18. $t = 0, \pi, 2\pi/3, 4\pi/3$ (4)

19. Without using your calculator, find the **exact value** of the expression. (7.6 #15c)

$\csc\left[\cos^{-1}\left(-\frac{1}{4}\right)\right]$

Let $\theta = \cos^{-1}\left(-\frac{1}{4}\right)$

$\cos\theta = -\frac{1}{4}$

$x = -1 \quad QII$

$y = \sqrt{15}$

$r = 4$

$\csc\theta = \frac{r}{y}, \csc\theta = \frac{4}{\sqrt{15}}$

19. $\frac{4}{\sqrt{15}}$ (4)

20. Use inverse trigonometric functions to find the solutions of $2\tan^2 t + 9\tan t + 3 = 0$ that are in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, and approximate the solutions to four decimal places. (7.6 #55)

$2\tan^2 t + 9\tan t + 3 = 0$

$\tan t = \frac{-9 \pm \sqrt{81 - 4(2)(3)}}{4} = \frac{-9 \pm \sqrt{57}}{4}$

$\tan t = \frac{-9 + \sqrt{57}}{4}, \quad \tan t = \frac{-9 - \sqrt{57}}{4}$

$t_R \approx 0.3478 \quad t_R \approx 1.3336$

tangent negative $\quad$ tangent negative

$t \approx -0.3478 \quad t \approx -1.3336$

20. $t \approx -0.3478, -1.3336$ (4)

21. In triangle ABC if $\alpha = 42°10'$, $\gamma = 61°20'$ and $b = 19.7$ find the value of **side** a. (8.1 #5)

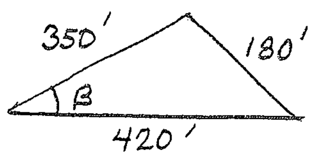
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad \beta = 180° - (\alpha + \gamma) = 76°30'$$

$$a = \frac{(19.7)(\sin 42°10')}{\sin 76°30'}$$

21. $a \approx 13.6$ (4)

$a \approx 13.6$

22. A triangular plot of land has sides of lengths 420 feet, 350 feet, and 180 feet. Approximate the smallest angle between the sides. (8.2 #18)



$\cos\beta \approx 0.90646$

$\beta \approx 24.979°$

$$\cos\beta = \frac{420^2 + 350^2 - 180^2}{2(420)(350)} \qquad \beta \approx 25°$$

22. $\approx 25°$ (4)

23. Approximate the area of a parallelogram that has sides of lengths a and b (in feet) if one angle at a vertex has measure θ.

$a = 12.0$ ft, $b = 16.0$ ft, $\theta = 40°$

Area of a triangle:

$A = \frac{1}{2}ab\sin\theta$ (8.2 #43)

$$A = 2\left(\frac{1}{2}(12)(16)(\sin 40°)\right)$$

$A \approx 123.4\ ft^2$

23. $A \approx 123.4\ ft^2$ (4)

Bonus: Find **all exact values** for the solutions of the equation. (7.2 #17)

$$\sin\left(2x - \frac{\pi}{3}\right) = \frac{1}{2} \qquad \text{Let } \theta = 2x - \frac{\pi}{3}$$

$$\sin\theta = \frac{1}{2} \qquad 2x - \frac{\pi}{3} = \frac{\pi}{6} + 2\pi n,\ 2x - \frac{\pi}{3} = \frac{5\pi}{6} + 2\pi n$$

$$\theta = \frac{\pi}{6} + 2\pi n \qquad 2x = \frac{\pi}{2} + 2\pi n,\ 2x = \frac{7\pi}{6} + 2\pi n \qquad x = \frac{\pi}{4} + \pi n,\ \frac{7\pi}{12} + \pi n$$

$$\theta = \frac{5\pi}{6} + 2\pi n \qquad x = \frac{\pi}{4} + \pi n,\ x = \frac{7\pi}{12} + \pi n$$

Bonus: ______ (4)