IUPUI Department of Mathematical Sciences
Departmental Final Examination

PRACTICE FINAL EXAM VERSION #2

MATH 15400

Trigonometry

Exam directions similar to those on the departmental final.

1. DO NOT OPEN this test booklet until you are told to do so.
2. This is NOT the exam for MATH 15300 or 15900.
3. There are 8 pages in this exam with problems 1 to 24 and a bonus problem.
4. You MUST get a new exam from the proctor if your exam is incomplete.
5. PRINT your name and student ID# below.
6. MARK your section below.
7. You will have two hours to complete this examination.
8. A TI-30Xa calculator is permitted, no other calculator is allowed.
9. No scrap paper, notes, books, or collaborators are allowed.
10. Exact answers may contain π or radicals or logarithms.
11. Simplify all answers completely.
12. Problems involving units must have the units represented on the answer to receive full credit.

Name (Print Clearly)	Solutions
Student ID#	

Practice Departmental Final Exam Recommendations to Students:

- Take this practice final exam like an actual examination (not like doing homework). That is, create an "exam like" atmosphere. This practice exam should be taken after completing a thorough review of the material.
- Set aside a two-hour block of time with no interruptions (no facebook, texting, phone calls, restroom breaks, etc.).
- Do not use any help aids, such as notes, textbook, internet, scrap paper, MAC staff, etc.
- Work through all problems noting which concepts you know well and which ones you need to spend more time on.
- Grade your exam using the answers in the back of your textbook (the textbook section and exercise number is noted at the top right of each problem).
- Rework any problem on the exam that you missed and then work similar problems from the textbook until you can perform the operations without error.
- Follow the same recommendations for taking the Practice Final Exam Version #1.

MATH 15400 Practice Departmental Final Exam (Version #2)

TEXTBOOK: Swokowski & Cole, *Algebra & Trigonometry with Analytic Geometry*, Classic 12th Edition

To receive full credit you must show all your work. Simplify all answers completely. Be sure to check your final answers for errors. Problems involving units must have the units represented on the answer to receive full credit.

1. Find an equation of the parabola with vertex $V(3,-5)$ and directrix $x=2$. (11.1 #23)

Horizontal $(y-k)^2=4p(x-h)$

$V(h,k)$ $x=h-p$

$V(3,-5)$ $2=h-p$ $(y+5)=4(1)(x-3)$

$2=3-p$ $(y+5)=4(x-3)$

$p=1$

1. $(y+5)=4(x-3)$ (4)

2. Find the vertices and the foci of the ellipse. Sketch its graph.

$4x^2+9y^2-32x-36y+64=0$ (11.2 #11) (4)

$4x^2-32x+9y^2-36y=-64$

$4(x^2-8x+16)+9(y^2-4y+4)=-64+64+36$

$4(x-4)^2+9(y-2)^2=36$

$\frac{(x-4)^2}{9}+\frac{(y-2)^2}{4}=1$

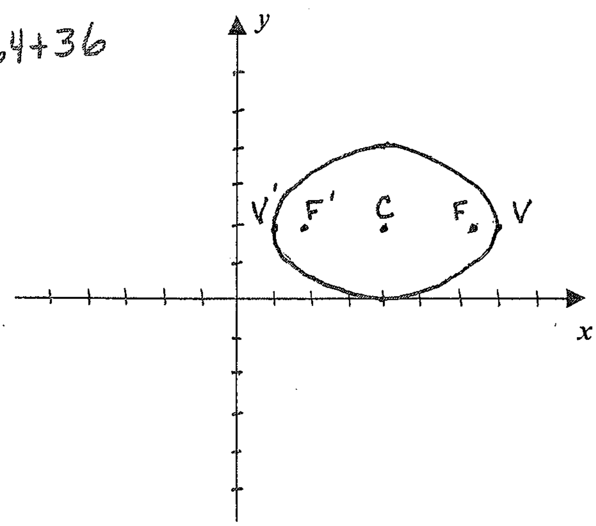
$C(4,2)$

$V(4\pm 3,2)$ $F(4\pm\sqrt{5},2)$

$M(4,2\pm 2)$

$c^2=a^2-b^2$

$c^2=5, c=\sqrt{5}$



3. Find an equation of the hyperbola that has its center at the origin with vertices $V(\pm 4,0)$ and passing through the point $(8,2)$. (11.3 #26)

Horizontal

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $a=4$ $a^2=16$ $x=8$ $y=2$ $\frac{x^2}{16}-\frac{y^2}{4/3}=1$

$\frac{64}{16}-\frac{4}{b^2}=1$ $\frac{-4}{b^2}=-3$ $\frac{x^2}{16}-\frac{3y^2}{4}=1$

$b^2=\frac{4}{3}$

3. $\frac{x^2}{16}-\frac{3y^2}{4}=1$ (4)

4. Solve the system. (9.1 #17)

$$\begin{cases} x^2+y^2=16 \\ 2y-x=4 \end{cases}$$

$x=2y-4$

$(2y-4)^2+y^2=16$

$4y^2-16y+16+y^2=16$

$5y^2-16y=0$

$y(5y-16)=0$

$y=0, \quad y=\frac{16}{5}$

$x=-4, \quad x=2\left(\frac{16}{5}\right)-4$

$x=\frac{12}{5}$

Two points $(-4,0), \left(\frac{12}{5}, \frac{16}{5}\right)$

4. $(-4,0), \left(\frac{12}{5}, \frac{16}{5}\right)$ (4)

5. ***Planning production*** A small furniture company manufactures sofas and recliners. Each sofa requires 8 hours of labor and $180 in materials, while a recliner can be built for $105 in 6 hours. The company has 340 hours of labor available each week and can afford to buy $6750 work of materials. How many recliners and sofas can be produced if all labor hours and all materials must be used? (9.2 #37)

Let $s=\#$ of sofas

$r=\#$ of recliners

$$\begin{cases} 8s+6r=340 \\ 180s+105r=6750 \end{cases}$$

$$\begin{cases} 4s+3r=170 \\ 12s+7r=450 \end{cases}$$

$-12s-9r=-510$

$12s+7r=450$

$-2r=-60$

$r=30$

$s=20$

5. 20 sofas, 30 recliners (4)

6. a) Calculate the length of arc that subtends a central angle of measure $\theta=50°$ on a circle of diameter 16 meters. (6.1 #35)

$s=r\theta, \; r=8\text{m}, \; \theta=50°\left(\frac{\pi}{180°}\right)=\frac{5\pi}{18}$

$s=8\left(\frac{5\pi}{18}\right)=\frac{20\pi}{9}\text{m}$

6a) $s=\frac{20\pi}{9}\text{m}$ (4)

b) Find the area of the sector determined by θ in part (a).

$A=\frac{1}{2}r^2\theta$

$A=\frac{1}{2}(8^2)\left(\frac{5\pi}{18}\right)=\frac{80\pi}{9}\text{m}^2$

6b) $A=\frac{80\pi}{9}\text{m}^2$ (4)

7. A wheel of radius 9 inches is rotating at a rate of 2400 rpm. (6.1 #46)

a) Find the angular speed (in radians per minute).

$$\omega = \frac{2400 \text{ rev.}}{\text{min}} \cdot \frac{2\pi \text{ radians}}{1 \text{ rev.}}$$

$$\omega = 4800\pi \text{ rad/min}$$

7a) $4800\pi \frac{\text{rad}}{\text{min}}$ (4)

b) Find the linear speed of a point on the circumference (in ft/min).

$$V = r\omega$$

$$V = \left(\frac{9\text{in}}{1}\right)\left(\frac{4800\pi}{\text{min}}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right) = 3600\pi \frac{\text{ft}}{\text{min}}$$

7b) $3600\pi \frac{\text{ft}}{\text{min}}$ (4)

8. Find the exact values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ if θ is in standard position and the terminal side of θ is in quadrant III and parallel to the line $2y - 7x + 2 = 0$. (6.2 #79)

$2y = 7x - 2$

$y = \frac{7}{2}x - 1$

$m = \frac{7}{2}$

Let $P(x, y)$ be $P(-2, -7)$

$x = -2$

$y = -7$

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{53}$

$\sin\theta = y/r$, $\sin\theta = \frac{-7}{\sqrt{53}}$

$\cos\theta = x/r$, $\cos\theta = \frac{-2}{\sqrt{53}}$

$\tan\theta = y/x$, $\tan\theta = \frac{7}{2}$

8. $\sin\theta = -\frac{7}{\sqrt{53}}$, $\tan\theta = \frac{7}{2}$, $\cos\theta = \frac{-2}{\sqrt{53}}$ (4)

9. Approximate, to the nearest 0.01 radian, all angles θ in the interval $[0, 2\pi)$ that satisfy the equation. (6.4 #37f)

$\csc\theta = -4.8521$

$\sin\theta = -\frac{1}{4.8521}$, $\theta_R \approx 0.20758$

$\theta_R \approx 0.21$

$\sin\theta < 0$ in QIII, QIV

QIII : $\theta = \pi + \theta_R$, $\theta \approx 3.35$

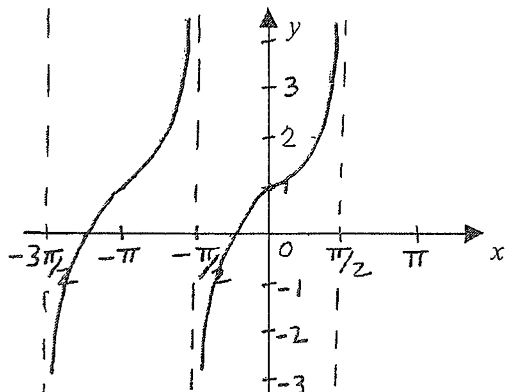
QIV : $\theta = 2\pi - \theta_R$, $\theta \approx 6.07$

9. $\theta \approx 3.35, 6.07$ (4)

10. Graph at least one complete period of $y = 1 + \tan x$. (6.3 #59)

(4)

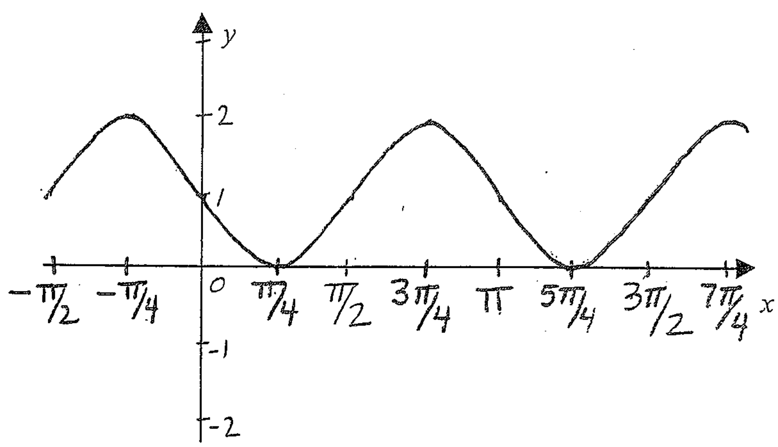
Shift $y = \tan x$
up 1 unit



11. Find the amplitude, period, phase shift, and graph at least one complete period for $y = \sin(2x - \pi) + 1$.

(6.5 #13)

(4)



amplitude = 1

period = π

$0 \le 2x - \pi \le 2\pi$

$\pi \le 2x \le 3\pi$

$\pi/2 \le x \le 3\pi/2$

phase shift

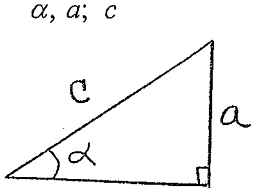
$x = \frac{\pi}{2}$

y-intercept $(0, 1)$

12. Given the indicated parts of triangle ABC with $\gamma = 90°$, express the third part in terms of the first two.

(6.7 #21)

α, a; c



$\sin\alpha = \frac{a}{c}$

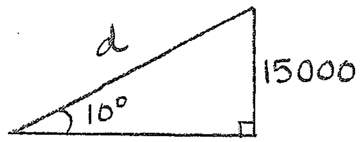
$c = \frac{a}{\sin\alpha}$ or

$c = a\csc\alpha$

$c = \frac{a}{\sin\alpha}$ or

12. $c = a\csc\alpha$ (4)

13. An airplane takes off at a 10° angle and travels at the rate of 250 ft/sec. Approximately how long does it take the airplane to reach an altitude of 15,000 feet? (6.7 #32)



$\sin 10^\circ = \frac{15000}{d}$

$d = \frac{15000}{\sin 10^\circ}$

$d = rt$

$t = \frac{d}{r}$

$t = \frac{\frac{15000}{\sin 10^\circ}}{250} = 60 \csc 10^\circ \text{ sec}$

$\approx 345.526 \text{ sec}$

$\approx 5.76 \text{ minutes}$

13. $t \approx 5.76$ min (4)

14. Verify the identity. **Show all work.** (7.1 #9) (4)

$$\frac{1}{1-\cos\gamma} + \frac{1}{1+\cos\gamma} = 2\csc^2\gamma$$

$$\frac{1}{1-\cos\gamma} + \frac{1}{1+\cos\gamma} = \frac{1+\cos\gamma+1-\cos\gamma}{(1-\cos\gamma)(1+\cos\gamma)}$$

$$= \frac{2}{1-\cos^2\gamma}$$

$$= \frac{2}{\sin^2\gamma}$$

$$= 2\csc^2\gamma$$

15. Find the **exact values** of the solutions of the equation that are in the interval $[0, 2\pi)$. (7.2 #55)

$2\tan t - \sec^2 t = 0$

$2\tan t - (1 + \tan^2 t) = 0$

$\tan^2 t - 2\tan t + 1 = 0$

$(\tan t - 1)(\tan t - 1) = 0$

$(\tan t - 1)^2 = 0$

$\tan t = 1$

$t = \pi/4, 5\pi/4$

15. $t = \pi/4, 5\pi/4$ (4)

16. If $\sin\alpha = -\frac{4}{5}$ and $\sec\beta = \frac{5}{3}$ for a third quadrant angle α and a first quadrant angle β, find the **exact value** for: (7.3 #21)

α: QIII $x=-3$, $y=-4$, $r=5$

β: QI $x=3$, $y=4$, $r=5$

a) $\sin(\alpha+\beta)$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \left(-\frac{4}{5}\right)\left(\frac{3}{5}\right) + \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)$$
$$= \frac{-12-12}{25} = \frac{-24}{25}$$

16a) $-\frac{24}{25}$ (4)

b) $\tan(\alpha+\beta)$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$
$$= \frac{\frac{4}{3}+\frac{4}{3}}{1-\left(\frac{4}{3}\right)\left(\frac{4}{3}\right)} = \frac{12+12}{9-16} = -\frac{24}{7}$$

16b) $-\frac{24}{7}$ (4)

17. Given $\sin\theta = -\frac{4}{5}$; $270° < \theta < 360°$ find the **exact value** of $\sin 2\theta$, $\cos 2\theta$ and $\tan 2\theta$. (7.4 #4)

$x=3$, $y=-4$, $r=5$ QIV

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2(-4/5)(3/5) = -24/25$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= (3/5)^2 - (-4/5)^2 = -7/25$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(-4/3)}{1-(-4/3)^2} = \frac{-24}{-7}$$
$$= \frac{24}{7}$$

17a) $\sin 2\theta$: $-\frac{24}{25}$ (4)

17b) $\cos 2\theta$: $\frac{-7}{25}$ (4)

17c) $\tan 2\theta$: $\frac{24}{7}$ (4)

18. Find the **exact values** of the solutions of the equation that are in the interval $[0, 2\pi)$. (7.4 #37)

$\cos u + \cos 2u = 0$

$\cos u + (2\cos^2 u - 1) = 0$

$2\cos^2 u + \cos u - 1 = 0$

$(2\cos u - 1)(\cos u + 1) = 0$

$\cos u = 1/2, \quad \cos u = -1$

$u = \pi/3, \ 5\pi/3, \quad u = \pi$

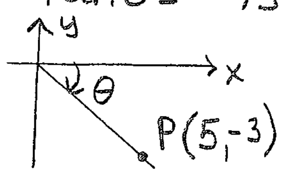
18. $u = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$ (4)

19. Without using your calculator, find the **exact value** of the expression, if it is defined. (7.6 #15b)

$\sec\left[\tan^{-1}\left(-\frac{3}{5}\right)\right]$

Let $\theta = \tan^{-1}(-3/5)$

$\tan\theta = -3/5$



$x = 5, \ y = -3, \ r = \sqrt{34}$

$\sec\theta = \frac{r}{x}$

$\sec\theta = \frac{\sqrt{34}}{5}$

19. $\frac{\sqrt{34}}{5}$ (4)

20. Use inverse trigonometric functions to find the solutions of $\cos^2 x + 2\cos x - 1 = 0$ that are in $[0, 2\pi)$, and approximate the solutions to four decimal places. (7.6 #53)

$\cos x = \frac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2} = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$

$\cos x = -1 - \sqrt{2}$, $\cos x = -1 + \sqrt{2}$ (positive)

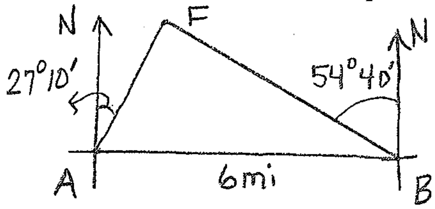
No solution $\quad x_R \approx 1.1437$

QI: $x = x_R \quad x \approx 1.1437$

QIV: $x = 2\pi - x_R, \ x \approx 5.1395$

20. $x \approx 1.1437, 5.1395$ (4)

21. A forest ranger at an observation point A sights a fire in the direction $N27°10'E$. Another ranger at an observation point B, 6.0 miles due east of A, sights the same fire at $N52°40'W$. Approximate the distance from observation point A to the fire. (8.1 #25)



$\angle FAB = 90° - 27°10' = 62°50'$

$\angle FBA = 90° - 54°40' = 37°20'$

$\angle AFB = 180° - 62°50' - 37°20' = 79°50'$

$\dfrac{\overline{AF}}{\sin 37°20'} = \dfrac{6}{\sin 79°50'}$ $\quad \overline{AF} = \dfrac{6\sin 37°20'}{\sin 79°50'}$ mi

$\overline{AF} \approx 3.70$ miles

21. ≈ 3.70 miles (4)

22. In triangle ABC if $\gamma = 115°10'$, $a = 1.10$ and $b = 2.10$ find the value of **side** c. (8.2 #9)

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = (1.10)^2 + (2.10)^2 - 2(1.10)(2.10)\cos 115°10'$

$c \approx 2.75$

22. $c \approx 2.75$ (4)

23. Use **Heron's formula** to approximate the area of triangle ABC.

$a = 25.0$ ft, $b = 80.0$ ft, $c = 60.0$ ft $\quad s = \frac{1}{2}(a+b+c)$, $A = \sqrt{s(s-a)(s-b)(s-c)}$ (8.2 #39)

$s = \frac{1}{2}(25+80+60) = 82.5$

$A = \sqrt{82.5(82.5-25)(82.5-80)(82.5-60)}$

$A = \sqrt{(82.5)(57.5)(2.5)(22.5)}$

$A \approx 516.56 \approx 516.6\ ft^2$

23. $A \approx 516.6\ ft^2$ (4)

Bonus: Find the **exact values** of the solutions of the equation that are in the interval $[0, 2\pi)$. (7.2 #39)

$\cos\left(2x - \frac{\pi}{4}\right) = 0$

$2x - \frac{\pi}{4} = \frac{\pi}{2} + n\pi$

$2x = \frac{3\pi}{4} + n\pi$

$x = \frac{3\pi}{8} + \frac{\pi}{2}n$

$n=0,\ x = \frac{3\pi}{8}$

$n=1,\ x = \frac{7\pi}{8}$

$n=2,\ x = \frac{11\pi}{8}$

$n=3,\ x = \frac{15\pi}{8}$

Bonus: $x = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$ (4)