IUPUI Department of Mathematical Sciences
Departmental Final Examination

PRACTICE FINAL EXAM VERSION #1

MATH 15300

College Algebra

Exam directions similar to those on the departmental final.

1. DO NOT OPEN this test booklet until you are told to do so.
2. This is NOT the exam for MATH 15400 or 15900.
3. There are 8 pages in this exam with problems 1 to 24 and a bonus problem.
4. You MUST get a new exam from the proctor if your exam is incomplete.
5. PRINT your name and student ID# below.
6. MARK your section below.
7. You will have two hours to complete this examination.
8. A TI-30Xa calculator is permitted, no other calculator is allowed.
9. No scrap paper, notes, books, or collaborators are allowed.
10. Exact answers may contain π or radicals or logarithms.
11. Simplify all answers completely.
12. Problems involving units must have the units represented on the answer to receive full credit.

Name (Print Clearly)	Solutions
Student ID#	

Practice Departmental Final Exam Recommendations to Students:

- Take this practice final exam like an actual examination (not like doing homework). That is, create an "exam like" atmosphere. This practice exam should be taken after completing a thorough review of the material.
- Set aside a two-hour block of time with no interruptions (no facebook, texting, phone calls, restroom breaks, etc.).
- Do not use any help aids, such as notes, textbook, internet, scrap paper, MAC staff, etc.
- Work through all problems noting which concepts you know well and which ones you need to spend more time on.
- Grade your exam using the answers in the back of your textbook (the textbook section and exercise number is noted at the top right of each problem).
- Rework any problem on the exam that you missed and then work similar problems from the textbook until you can perform the operations without error.
- Follow the same recommendations for taking the Practice Final Exam Version #2.

MATH 15300 Departmental Practice Final Exam (Version #1)

TEXTBOOK: Swokowski & Cole, *Algebra & Trigonometry with Analytic Geometry*, Classic 12th Edition

To receive full credit you must show all your work. Simplify all answers completely. Be sure to check your final answers for errors. Problems involving units must have the units represented on the answer to receive full credit.

1. Simplify. (1.2 #29)

$$\left(5x^2y^{-3}\right)\left(4x^{-5}y^4\right)$$

$$= \left(\frac{5x^2}{y^3}\right)\left(\frac{4y^4}{x^5}\right) = \frac{20y}{x^3}$$

1. $= \dfrac{20y}{x^3}$ (4)

2. Factor the polynomial. (1.3 #97)

$$y^2 - x^2 + 8y + 16$$

$$= y^2 + 8y + 16 - x^2$$

$$= (y+4)^2 - x^2$$

$$= (y+4-x)(y+4+x)$$

2. $= (y+4-x)(y+4-x)$ (4)

3. Simplify the expression. (1.4 #37)

$$\frac{y^{-1} + x^{-1}}{(xy)^{-1}}$$

LCD: xy

$$= \frac{\frac{1}{y} + \frac{1}{x}}{\frac{1}{xy}} = x + y$$

3. $= x + y$ (4)

4. Solve for the specified variable. (2.1 #73)

$$S = \frac{p}{q + p(1-q)} \quad \text{for } q$$

$$S(q+p-pq) = p$$

$$Sq + Sp - Spq = p$$

$$Sq - Spq = p - Sp$$

$$q(S - Sp) = p - Sp \nearrow$$

$$q = \frac{p - Sp}{S - Sp} \text{ or}$$

$$q = \frac{p(1-S)}{S(1-p)}$$

4. $q = \dfrac{p(1-S)}{S(1-p)}$ (4)

5. ***Movie attendance*** Six hundred people attended the premiere of a motion picture. Adult tickets cost \$9, and children were admitted for \$6. If box office receipts totaled \$4800, how many children attended the premiere?

(2.2 #11)

Let $x =$ # of adult tickets

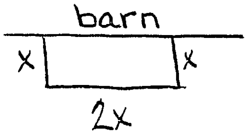
$600 - x =$ # of children tickets

$$9x + 6(600-x) = 4800$$
$$9x + 3600 - 6x = 4800$$
$$3x = 1200$$ 400 adult tickets
$$x = 400$$ 200 children tickets

5. 200 children (4)

6. ***Fencing a region*** A farmer plans to enclose a rectangular region, using part of his barn for one side and fencing for the other three sides. If the side parallel to the barn is to be twice the length of an adjacent side, and the area of the region is to be 128 ft^2, how many feet of fencing should be purchased?

(2.3 #66)



$A = lw$, $A = 128\ ft^2$

$A = (2x)(x)$

$2x^2 = 128$

$x^2 = 64$

$x = \pm 8$, length is positive

$x = 8$

Feet of fencing

$8 + 16 + 8 = 32$ ft

6. 32 feet (4)

7. Write the expression in the form $a + bi$, where a and b are real numbers.

(2.4 #23)

$$\frac{-4+6i}{2+7i} \quad \frac{-4+6i}{2+7i} \cdot \frac{2-7i}{2-7i}$$
$$= \frac{-8+28i+12i-42i^2}{2^2+7^2}$$
$$= \frac{34+40i}{53} = \frac{34}{53} + \frac{40}{53}i$$

7. $= \frac{34}{53} + \frac{40}{53}i$ (4)

8. Solve for the specified variable.

(2.5 #55)

$S = \pi r\sqrt{r^2+h^2}$ for h

$$\frac{S}{\pi r} = \sqrt{r^2+h^2}$$
$$\frac{S^2}{\pi^2 r^2} = r^2 + h^2$$
$$h^2 = \frac{S^2}{\pi^2 r^2} - r^2 \nearrow$$
$$h^2 = \frac{S^2 - \pi^2 r^4}{\pi^2 r^2}$$
$$h = \pm\frac{\sqrt{S^2-\pi^2 r^4}}{\pi r}$$
$$h = \frac{\sqrt{S^2-\pi^2 r^4}}{\pi r}, \ h>0$$

8. $h = \frac{\sqrt{S^2-\pi^2 r^4}}{\pi r}$, $h > 0$ (4)

9. Solve the inequality, and **express the solutions in terms of intervals** whenever possible. (2.6 #33)

$$4 > \frac{2-3x}{7} \geq -2$$

$$28 > 2-3x \geq -14$$

$$26 > -3x \geq -16$$

$$-\frac{26}{3} < x \leq \frac{16}{3}$$

$$\left(-{}^{26}/_3, {}^{16}/_3\right]$$

9. $\left(-\frac{26}{3}, \frac{16}{3}\right]$ (4)

10. Solve the inequality. Express the solution in **interval notation**. (2.7 #37)

$$\frac{x}{3x-5} \leq \frac{2}{x-1}$$

$$\frac{x}{3x-5} - \frac{2}{x-1} \leq 0$$

$$\frac{x(x-1)-2(3x-5)}{(3x-5)(x-1)} \leq 0$$

$$\frac{x^2-x-6x+10}{(3x-5)(x-1)} \leq 0$$

$$\frac{x^2-7x+10}{(3x-5)(x-1)} \leq 0 \uparrow$$

$$\frac{(x-2)(x-5)}{(3x-5)(x-1)} \leq 0$$

zeros: $x=2,5$

Division by zero: $x=\frac{5}{3}, 1$

−	−	−	+	+	$(x-2)$
−	−	−	−	+	$(x-5)$
−	−	+	+	+	$(3x-5)$
−	+	+	+	+	$(x-1)$
+	−	+	−	+	Quotient $\frac{(x-2)(x-5)}{(3x-5)(x-1)}$
1	5/3	2	5		

10. $(1, {}^5/_3) \cup [2,5]$ (4)

11. Given the points $A(4,-3)$ and $B(6,2)$. (3.1 #9)

(a) Find the distance $d(A,B)$ between A and B.

$$d(A,B) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d = \sqrt{(6-4)^2+(2+3)^2}$$

$$d = \sqrt{4+25} = \sqrt{29}$$

11(a) $d(A,B) = \sqrt{29}$ (2)

(b) Find the midpoint of the segment AB.

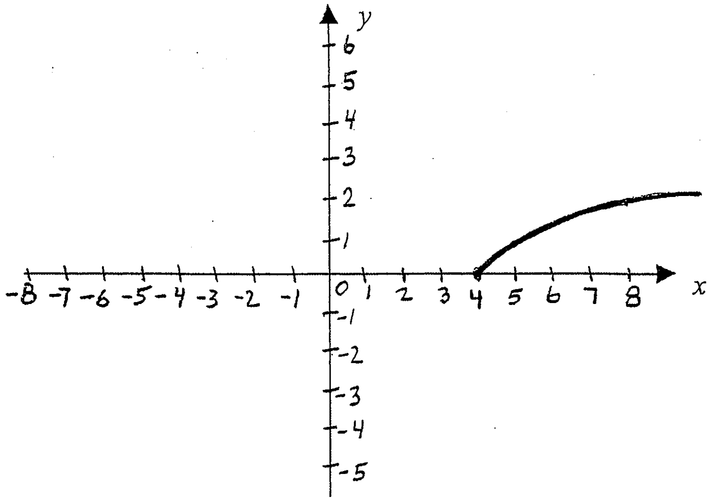
$$M_{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M_{AB} = \left(5, -{}^1/_2\right)$$

11(b) $\left(5, -{}^1/_2\right)$ (2)

12. Sketch the graph $y = \sqrt{x-4}$, and label the x- and y-intercepts. (3.2 #20)

(4)



13. For the given circle, find the x-intercepts. (3.2 #67)

$x^2 + y^2 - 4x - 6y + 4 = 0$

To find x-intercepts, set $y=0$.

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

x-intercept $(2,0)$

13. $(2,0)$ (4)

14. Given the points $A(3,-1)$ and $B(-2,6)$. Find an equation (in slope-intercept form) for the perpendicular bisector of segment AB.

(3.3 #37)

$m_{AB} = \dfrac{6-(-1)}{-2-3} = \dfrac{7}{-5}$, $m_{\perp} = \dfrac{5}{7}$, $M_{AB} = (1/2, 5/2)$

$y - y_1 = m(x - x_1)$

$y - 5/2 = 5/7(x - 1/2)$

$y = \frac{5}{7}x - \frac{5}{14} + \frac{5}{2}$

$y = \frac{5}{7}x + \frac{30}{14}$, $y = \frac{5}{7}x + \frac{15}{7}$

14. $y = \frac{5}{7}x + \frac{15}{7}$ (4)

15. ***Childhood growth*** For children between ages 6 and 10, height y (in inches) is frequently a linear function of age t (in years). The height of a certain child is 48 inches at age 6 and 50.5 inches at age 7. (3.4 #71)

(a) Express y as a function of t.

t	y
6	48
7	50.5

$m = \frac{50.5-48}{7-6} = 2.5$

$y - 48 = 2.5(t-6)$

$y = 2.5t + 33$

15(a) $y = 2.5t + 33$ (2)

(b) Predict the height of the child at age 10.

$y = 2.5(10) + 33$

$y = 58$ inches

15(b) 58 inches (2)

16. Determine whether f is even, odd, or neither even nor odd. (3.5 #4)

$f(x) = |x| - 3$

$f(-x) = |-x| - 3$

$= |x| - 3$

$= f(x)$

$f(x)$ is an even function

16. $f(x)$ is even (4)

17. ***Height of a projectile*** An object is projected vertically upward from the top of a building with an initial velocity of 144 ft/sec. Its distance $s(t)$ in feet above the ground after t seconds is given by the equation $s(t) = -16t^2 + 144t + 100$. (3.6 #41)

(a) Find its maximum distance above the ground.

Max. occurs at $t = -b/2a$. $t = \frac{-144}{2(-16)} = \frac{9}{2}$ sec.

$s(9/2) = -16(9/2)^2 + 144(9/2) + 100$

$= -324 + 648 + 100 = 424$ feet

17(a) 424 feet (4)

(b) Find the height of the building.

$t = 0$

$s(0) = 100$ feet

17(b) 100 feet (2)

18. Given $f(x) = \frac{2x}{x-4}$ and $g(x) = \frac{x}{x+5}$. (3.7 #7)

(a) Find $(f/g)(x)$.

$(f/g)(x) = \frac{2x}{x-4} \div \frac{x}{x+5} = \frac{2x}{x-4} \cdot \frac{x+5}{x}$

$(f/g)(x) = \frac{2(x+5)}{x-4}$

18(a) $\left(\frac{f}{g}\right)(x) = \frac{2(x+5)}{x-4}$ (2)

(b) Find the domain of $(f/g)(x)$.

$x \neq 4, -5, 0$

$D = \{x \mid x \in \mathbb{R}, x \neq -5, 0, 4\}$

18(b) $\{x \mid x \in \mathbb{R}, x \neq -5, 0, 4\}$ (2)

19. Given $f(x) = x^3 + 2x^2 - 4x - 8$. (4.1 #23)

(a) Find all values of x such that $f(x) > 0$ and $f(x) < 0$.

$x^3 + 2x^2 - 4x - 8 = 0$

$x^2(x+2) - 4(x+2) = 0$

$(x+2)(x^2-4) = 0$

$(x+2)(x+2)(x-2) = 0$

$(x+2)^2(x-2) = 0$

zeros: $x = -2, -2, 2$

+	+	+	$(x+2)^2$
−	−	+	$(x-2)$
−	−	+	$f(x)$
-2	2		

$f(x) > 0: (2, \infty)$

$f(x) < 0: (-\infty, -2) \cup (-2, 2)$

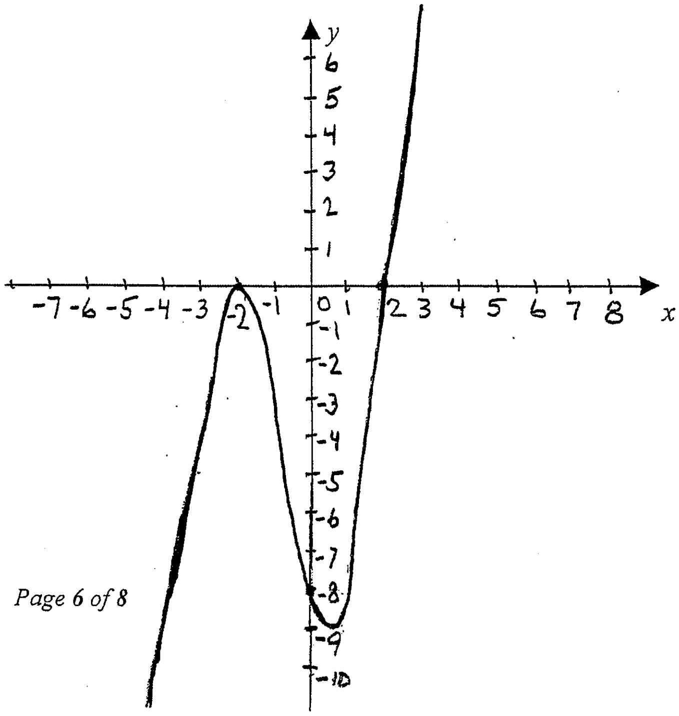
19(a) (4)

(b) Sketch the graph of $f(x)$. (4)

$f(x) < 0: (-\infty, -2) \cup (-2, 2)$

$f(x) > 0: (2, \infty)$

x	y
-3	-5
-1	-3
0	-8
1	-9
3	25



20. Find the quotient and the remainder if $3x^5+6x^2+7$ is divided $x+2$. (4.2 #25)

$$\begin{array}{r l}
 & 3x^4-6x^3+12x^2-18x+36 \\
x+2 \,\big) & 3x^5+0x^4+0x^3+6x^2+0x+7 \\
 & -(3x^5+6x^4) \\
 & \quad -6x^4 \\
 & \quad -(-6x^4-12x^3) \\
 & \qquad 12x^3+6x^2 \\
 & \qquad -(12x^3+24x^2) \\
 & \qquad\quad -18x^2 \\
 & \qquad\quad -(-18x^2-36x) \\
 & \qquad\qquad 36x+7 \\
 & \qquad\qquad -(36x+72) \\
 & \qquad\qquad -65
\end{array}$$

Check $f(-2)=-65$ ✓
Remainder Theorem

OR

$$\begin{array}{r|rrrrrr}
-2 & 3 & 0 & 0 & 6 & 0 & 7 \\
 & & -6 & 12 & -24 & 36 & -72 \\
\hline
 & 3 & -6 & 12 & -18 & 36 & -65
\end{array}$$

$q(x)=3x^4-6x^3+12x^2-18x+36$

$r(x)=-65$

20. ______________ (4)

21. Given the one-to-one function $f(x)=\sqrt[3]{x}+1$, find the inverse function, $f^{-1}(x)$. (5.1 #37)

$y=\sqrt[3]{x}+1$

$\sqrt[3]{x}=y-1$

$x=(y-1)^3$

$y=(x-1)^3$

$f^{-1}(x)=(x-1)^3$

Check

$f(f^{-1}(x))=f((x-1)^3)$

$=\sqrt[3]{(x-1)^3}+1$

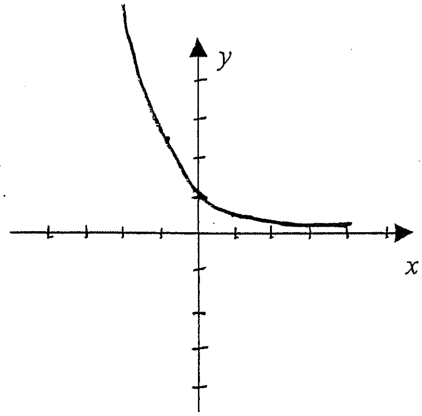
$=(x-1)+1$

$=x$ ✓

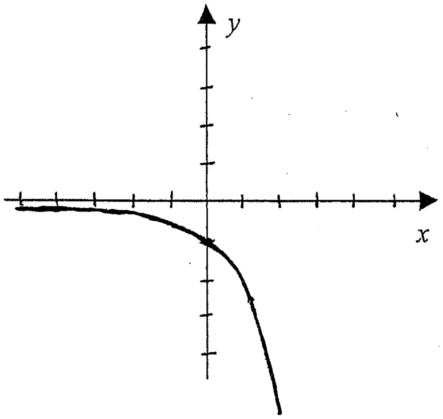
21. $f^{-1}(x)=(x-1)^3$ (4)

22. Sketch the graphs and dash in the asymptotes. (5.3 #1)

a) Graph $f(x)=e^{-x}$. (2)



b) Graph $f(x)=-e^{x}$. (2)



23. ***Radioactive iodine decay*** Radioactive iodine ^{131}I is frequently used in tracer studies involving the thyroid gland. The substance decays according to the $A(t)=A_0a^{-t}$, where A_0 is the initial dose and t is the time in days. Find a, assuming the half-life of ^{131}I is 8 days. (5.4 #73)

Find a when $A(t)=\frac{1}{2}A_0$, $t=8$

$\frac{1}{2}A_0=A_0a^{-t}$ $\quad \frac{1}{2}=\frac{1}{a^8}$

$\frac{1}{2}=a^{-8}$ $\quad a^8=2$

$a=\sqrt[8]{2}$ OR $2^{1/8}$

23. $a=2^{1/8}$ (4)

24. Find the **exact value** for the solution of the equation. (5.6 #19)

$\log(x^2+4)-\log(x+2)=2+\log(x-2)$

$\log\frac{x^2+4}{x+2}-\log(x-2)=2$

$\log\frac{x^2+4}{(x+2)(x-2)}=2$

$\frac{x^2+4}{x^2-4}=10^2$ ↗

$x^2+4=100x^2-400$

$99x^2=404$

$x^2=\frac{404}{99}$

$x=\pm\sqrt{\frac{404}{99}}=\pm\frac{2}{3}\sqrt{\frac{101}{11}}$ ↑

$x=-\frac{2}{3}\sqrt{\frac{101}{11}}$ Extraneous

24. $x=\frac{2}{3}\sqrt{\frac{101}{11}}$ (4)

Bonus: If \$1000 is invested at a rate of 7% per year compounded monthly, find the balance after 6 months. (5.2 #41)

$A=P(1+r/n)^{nt}$ $\quad t=\frac{1}{2}$ yr.

$A=1000\left(1+\frac{0.07}{12}\right)^{12(1/2)}$

$A=\$1000\left(1+\frac{0.07}{12}\right)^{6}\approx\$1035.51$

Bonus: $\approx\$1035.^{\underline{51}}$ (4)