

# MATH 16600 Final Exam

Exam is 7 pages plus cover page. Follow the instructions for each question. Show enough of your work that we can understand what you are doing.

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**1** (12 points) If  $f(x) = \ln \sqrt{1 + x^4}$ , find  $f'(x)$ .

**2** (12 points) The half-life of a radioactive substance is 100 years. A sample of the substance has a mass of 80 mg. Find a formula for the mass of the sample that remains after  $t$  years. When will the mass be reduced to 10 mg?

**3** (12 points) Find the limit.  $\lim_{x \rightarrow 0} \frac{\ln(1+x) - x}{x^2}$ .

4 (14 points) Evaluate the integral  $\int \frac{4}{x^2 - 4} dx$

5 (12 points) Evaluate the integral.  $\int (2 \cos^2 x - 1) \sin x dx$ .

6 (12 points) Evaluate the integral.  $\int 3x^2 \ln x dx$ .

**7** (12 points) Set up an integral that represents the length of the curve  $x = \sqrt{y} - y$ ,  $1 \leq y \leq 4$ .

**8** (12 points) Given  $f(x) = 1 + 5x + x^5$ , find  $f^{-1}(1)$  and  $(f^{-1})'(1)$ .

**9** (12 points) Find an equation of the tangent line to the curve at the point corresponding to the given value of the parameter.  
 $x = 1 + 2t$ ,  $y = t^3 - 2t$ ;  $t = 1$ .

**10** (12 points) Given a polar equation  $r = 2 - \cos \theta$ . Sketch the curve and find the area that it encloses.

**11** (14 points) Determine whether  $\int_2^{\infty} \frac{1}{x\sqrt{\ln x}} dx$  is convergent or divergent. Evaluate the integral if it is convergent.

**12** (12 points)  $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ . Determine whether the series is convergent or divergent.

**13** (12 points) Test the series for convergence or divergence.  $\sum_{n=1}^{\infty} \frac{3^n}{2^n+4^n}$ .

**14** (12 points) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.  $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n^2}$ .

**15** (14 points) Test the series for convergence or divergence. Find the sum if it is convergent.  $\sum_{n=1}^{\infty} \frac{9}{10^n}$ .

**16** (14 points) Find the radius of convergence and interval of convergence of the series.  $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ .

**17** (8 points, Bonus) Set up an integral that represents the area of the surface obtained by rotating the curve  $y = 1 + 2x^2$ ,  $1 \leq x \leq 2$ , about the  $y$ -axis.

**18** (8 points, Bonus) Find the Taylor series at  $a = 1$  for the function  $f(x) = e^{3x}$ .