

MATH 16600 Final Exam

Exam is 7 pages plus cover page. Follow the instructions for each question. Show enough of your work that we can understand what you are doing.

1 (14 points) Find the derivative of f

(a) $f(x) = \sin^{-1}(e^{2x})$

(b) $f(x) = x \ln(\sqrt{x})$

2 (8 points) Find the points on the curve $y = x^2e^{-x}$ at which the tangent line is horizontal.

3 (8 points) Determine if the following integral is convergent or divergent and if it is convergent find its value

$$\int_1^{\infty} \frac{1}{x^2 + x} dx$$

4 (21 points) Evaluate

(a) $\int x \ln(x) dx$

(b) $\int \frac{x^2}{\sqrt{1-x^2}} dx$

(c) $\int [\cos x - \cos^3 x] dx$

5 (8 points) Given $f(x) = 1 - 2x - \sin x$. Find $(f^{-1})'(1)$.

6 (8 points) Find the area of the surface obtained by rotating the curve $y = 1 + 2x^2$, $1 \leq x \leq 2$, about the y -axis.

7 (8 points) Sketch the polar curve $r = 3 - \sin \theta$ and find the area that it encloses.

8 (9 points) Find the Taylor series for $f(x) = e^{-x}$ at $a = 1$ and determine the radius of convergence of the series.

9 (8 points) Test the series for convergence or divergence:

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^3 + 1}$$

Don't forget to say which test are you using.

10 (8 points) Find the radius of convergence and the interval of convergence of the series. If the interval is finite do not forget to test the end points.

$$\sum_{n=1}^{\infty} \frac{2^n}{n!} (x - 1)^n.$$

Bonus

11 (4 points) Test the following series for convergence or divergence:

$$\sum_{n=1}^{\infty} \frac{4^n}{3^n + 5^n}$$

12 (4 points) Find the exact length of the polar curve: $r = 2(1 + \cos \theta)$. Hint: $\int_0^{2\pi} \sqrt{1 + \cos \theta} \, d\theta = 4\sqrt{2}$