

MATH 16600 Final Exam

Exam is 7 pages plus cover page. Follow the instructions for each question. Show enough of your work that we can understand what you are doing.

1 (12 points) If $f(x) = \ln \left[\frac{(2x+1)^4}{\sqrt{x+1}} \right]$, find $f'(x)$.

2 (12 points) A bacteria culture initially contains 80 cells and grows at a rate proportional to its size. After an hour the population has increased to 240. Find an expression for the number of bacteria after t hours.

3 (12 points) Find the limit. $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$.

4 (14 points) Evaluate the integral $\int \frac{5}{(x-3)(x+2)} dx$

5 (12 points) Evaluate the integral. $\int \cos^2 x \sin^3 x dx$.

6 (12 points) Evaluate the integral. $\int x \ln x dx$.

7 (12 points) Set up an integral that represents the length of the curve $y = \frac{x^2}{2} - \ln x$, $1 \leq x \leq 4$.

8 (12 points) Set up an integral that represents the area of the surface obtained by rotating the curve $y = x^2$, $0 \leq x \leq 2$, about the y -axis.

9 (12 points) Sketch the curve and find the area that it encloses. $r = \sqrt{1 + \sin \theta}$.

10 (12 points) Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.
 $x = t^3 - 2t$, $y = t + 3$; $t = 1$.

11 (14 points) Determine whether $\int_2^{\infty} \frac{1}{x(\ln x)^2} dx$ is convergent or divergent. Evaluate the integral if it is convergent.

12 (12 points) $\sum_{n=1}^{\infty} \frac{n-1}{n+1}$. Determine whether the series is convergent or divergent.

13 (12 points) Test the series for convergence or divergence. $\sum_{n=1}^{\infty} \frac{n!}{2^{n^2}}$.

14 (12 points) Determine whether the series is absolutely convergent, conditionally convergent, or divergent. $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$.

15 (14 points) Test the series for convergence or divergence. Find the sum if it is convergent. $\sum_{n=1}^{\infty} (-1)^n 2^{-n}$.

16 (14 points) Find the radius of convergence and interval of convergence of the series. $\sum_{n=1}^{\infty} \frac{x^n}{n 2^n}$.

17 (8 points, Bonus) Given $f(x) = 1 + 2x + x^3$, find $f^{-1}(1)$ and $(f^{-1})'(1)$.

18 (8 points, Bonus) Find the Taylor series at $a = 1$ for the function $f(x) = \ln(4 - x)$.