

# MATH 16600 Final Exam

Exam is 7 pages plus cover page. Follow the instructions for each question. Show enough of your work that we can understand what you are doing.

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- 1 (14 points) Find the derivative of  $f$

(a)  $f(x) = \cos^{-1}(\ln(x))$

(b)  $f(x) = \ln(e^x + 1)$

**2** (8 points) Find the exact length of the curve  $y = \frac{1}{8}x^2 - \ln x$ ,  $1 \leq x \leq 2$ .

**3** (8 points) Determine if the following integral is convergent or divergent and if it is convergent find its value

$$\int_2^{\infty} \frac{1}{x(\ln x)^3} dx$$

4 (21 points) Evaluate

(a)  $\int \frac{x^2 + x + 1}{x + 1} dx$

(b)  $\int \cos(2x) \cos x dx$

(c)  $\int \frac{\ln x}{x^2} dx$

**5** (8 points) Let  $f(x) = x^3 + x + 1$ . Find  $(f^{-1})'(1)$ .

**6** (8 points) Sketch the curve and find the exact area of the region enclosed by the polar curve.  $r = 3 - 2 \cos \theta$ .

7 (9 points) Find the Taylor series for  $f(x) = \frac{1}{7-2x}$  centered at  $a = 3$  and determine the associated radius of convergence.

8 (8 points) Test the series for convergence or divergence:

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n + \sqrt{n}}$$

Don't forget to say which test are you using.

**9** (8 points) Determine whether the series is convergent or divergent.  $\sum_{n=0}^{\infty} (-1)^n \frac{\cos n}{n^2}$ . Don't forget to say which test are you using.

**10** (8 points) Find the radius of convergence and the interval of convergence of the series. If the interval is finite do not forget to test the end points.

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{3^n \cdot n!}.$$

**Bonus**

**11** (4 points) Set up an integral that represents the area of the surface obtained by rotating the curve  $x = 2t^2 + 1/t$ ,  $y = 8\sqrt{t}$ ,  $1 \leq t \leq 3$  about the  $x$ -axis.

**12** (4 points) Find a power series representation  $\sum_{n=0}^{\infty} c_n x^n$  for the function and determine the radius of convergence.  
 $f(x) = \frac{1}{(1-x)^2}$ .