

MATH 16500 Final Exam

Exam is 7 pages plus cover page. Follow the instructions for each question. Show enough of your work that we can understand what you are doing.

1 (12 points) Determine the infinite limit and show your work. $\lim_{x \rightarrow 1^-} \frac{x^2 - x - 2}{x - 1}$.

2 (12 points) Evaluate the limit, if it exists. $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3}$.

3 (12 points) Given $f(x) = \frac{1}{x}$, find $f'(a)$ using $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$. No credits if you use any other formulas.

4 (12 points) Differentiate $f(\theta) = \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}$ with respect to θ .

5 (12 points) Find the derivative of $y = \frac{2 + 3x}{\sqrt{x}}$ with respect to x .

6 (12 points) Given $y \cos x - x \cos y = y^3 + 1$. Find dy/dx by implicit differentiation.

7 (12 points) Find the linearization $L(x)$ of the function $f(x) = \frac{1}{x}$ at $a = 1$.

8 (14 points) Let $f(x) = x^{3/5}(8 - x)$. Find the critical numbers of f .

9 (14 points) Given $y = \frac{1}{x^2 - 1}$. Sketch the graph indicating the local maximum/minimum points, and points of inflection.

10 (14 points) A circular cylindrical tank with an open top has a volume of 1000π cubic units. Find the smallest possible surface area of the tank.

11 (12 points) Express the integral as a limit of Riemann sums, $\int_0^\pi \sqrt{\sin x} \cos x \, dx$.

12 (12 points) Evaluate the integral $\int_0^1 3x\sqrt{1+x^2} \, dx$.

13 (12 points) Evaluate the indefinite integral $\int \frac{1+x}{\sqrt{x}} \, dx$.

14 (14 points) Sketch the region enclosed by the given curves and find the area. $y = x^2$, $y = \frac{3}{4}x^2 + 1$.

15 (12 points) Set up an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = x^3$, $y = 1$ and $x = 0$ about the y -axis.

16 (12 points) A circular swimming pool has a diameter of 30 *ft* the sides are 5 *ft* high, and the depth of the water is 4 *ft*. How much work is required to pump all of the water out over the side? (Use the fact that water weighs 62.5 *lb/ft*³.)

17 (Bonus, 8 points) Prove the statement using the ϵ, δ definition of a limit. $\lim_{x \rightarrow 2} \frac{x+4}{3} = 2$.

18 (Bonus, 8 points) Use the method of cylindrical shells to set up an integral for the volume generated by rotating the region bounded by $y = x(1-x)$ and $y = 0$ about the line $x = 1$.