MATH 16500 Final Exam — Spring 2017

Exam is 10 pages plus cover page. Follow the instructions for each question. Show enough of your work that we can understand what you are doing.

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1 (4 points) Prove the statement using the ϵ, δ definition of a limit. $\lim_{x \to 1} \frac{5x-1}{4} = 1$.

2 (6 points) Let $f(x) = x^{-1/2} + \frac{1}{16}x$. Find the absolute maximum value and the absolute minimum value of f on $[1, 9]$.

3 (12 points) Find the limit if it exists. Explain why if it does not exist. You must show your work to get full credit for an correct answer.

(a) $\displaystyle\lim_{x\to 2^-} \frac{x^2+x-2}{x^2-x-2}$.

(b) $\displaystyle\lim_{x\to 16} \frac{4-\sqrt{x}}{16x-x^2}$

(c) $\displaystyle\lim_{h\to 0} \frac{(2+h)^{-1}-2^{-1}}{h}$.

4 (12 points) Compute the derivatives of the following functions.

(a) $f(x) = \sqrt{x}(1+x)^2$

(b) $f(x) = (x^2+1)^{2017}(x^2-1)^{2018}$

(c) $f(x) = \displaystyle\int_1^{\frac{1}{x}} \frac{t}{(t^2+1)^2}\,dt$

5 (6 points) Use implicit differentiation to find an equation of the tangent line to the curve $\sqrt{y^3 + x} = x^2 y + x$ at $(1, 2)$.

6 (6 points) Sketch the region enclosed by the curves: $y = x^2 - 3,\ y = -2x$. Find the area of the region.

7 (12 points) Let $f(x) = -x^3 + 3x^2 + 9x$.

(a) find critical numbers,

(b) determine intervals on which f is increasing or decreasing,

(c) find local maximum values and local minimum values,

(d) determine the intervals on which the graph is concave up or concave down,

(e) Sketch the graph.

8 (6 points) A rectangular box has volume of 72 cubic units. Assume that the length of the base is twice the width, find the dimensions that will minimize the surface area.

9 (6 points) Express the integral as a limit of Riemann sums. Do not evaluate the limit. $\int_1^2 \sqrt{x^2+1}\, dx$.

10 (12 points) Evaluate the following integrals:

(a) $\displaystyle\int_0^3 (x+5)(x-1)dx.$

(b) $\displaystyle\int \frac{\sqrt{x}-2x}{x^3}\, dx.$

(c) $\displaystyle\int \frac{\sin x}{\sqrt{\cos x}}\, dx.$

11 (6 points) Let A be the region bounded by the graphs of $y = x^2$ and $y = -x$. Set up (but do not compute) the integral to find the volume of the solid generated by revolving A about the y-axis. Indicate the methods you use.

12 (6 points) Find the average value of $f(x) = \dfrac{(1+\sqrt{x})^2}{\sqrt{x}}$ on $[0, 9]$.

13 (6 points) The height of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 4 cm^2/min. At whate rate is the base of the triangle changing when the altitude is 10 cm and the area is 50 cm^2?

Bonus

14 (4 points). Set up an integral to find the volume of the solid generated by rotating the region bounded by $y = 2x$, $y = x^2 - 3$ about $y = -3$.

15 (4 points) Evaluate $\displaystyle\int \frac{x}{(x-1)^2} dx$.