

MATH 16500 Final Exam

Exam is 10 pages plus cover page. Follow the instructions for each question. Show enough of your work that we can understand what you are doing.

1 (4 points) State the limit definition of derivative. Use this definition to show that the derivative of $f(x) = x^2 + 3x$ is $f'(x) = 2x + 3$.

2 (8 points) Let $f(x) = x^{1/3}(4 - x)$. Find the absolute maximum value and the absolute minimum value of f on $[0, 8]$.

3 (12 points) Find the limit. If it is $+\infty$ or $-\infty$, say so. Show your work.

(a) $\lim_{x \rightarrow \infty} (\sqrt{x^2 - x} - x)$.

(b) $\lim_{x \rightarrow -1} \frac{x^2 - 3x - 4}{x^2 + 5x + 4}$

(c) $\lim_{x \rightarrow 0} \frac{\sin(2x)}{x^2 + 2x}$

4 (12 points) Compute the derivatives of the following functions.

(a) $f(x) = \frac{\sin x}{1 - \sin x}$

(b) $f(x) = (2x + 1)(x^2 + 1)$

(c) $f(x) = \cos(x^2 + 1)$

5 (8 points) Use implicit differentiation to find an equation of the tangent line to the curve $3xy^3 + y^2 = y^2x + 3$ at $(1, 1)$.

6 (8 points) Sketch the region enclosed by the curves: $y = \sqrt{x}$ and $y = x^2$. Find the area of the region.

7 (12 points) Let $f(x) = 1 + \frac{x-2}{x^2}$.

- (a) find the x-intercepts,
- (b) determine asymptotes,
- (c) find critical numbers,
- (d) determine intervals on which f is increasing or decreasing,
- (e) find local maximum values and local minimum values,
- (f) determine the intervals on which the graph is concave up or concave down,
- (g) Sketch the graph.

8 (8 points) A rectangular box in dimensions $a \times b \times c$ has volume of $4/3$ cubic units. Find the dimensions that will minimize the surface area if $b = 3a$.

9 (4 points) Estimate the integral $\int_0^4 (x^2 - 1) dx$ using a Riemann sum with $n = 4$ and right endpoints.

10 (12 points) Evaluate the following integrals:

(a) $\int_0^1 x^{1/3}(4 - 7x) dx.$

(b) $\int_0^1 6x^2(x^3 + 1)^2 dx.$

(c) $\int_1^9 \frac{\sqrt{x} + 4x^2}{x} dx.$

11 (12 points) Let R be the region bounded by the graphs of $y = \sqrt{x}$, $y = 1$ and $x = 0$. Set up (but do not compute) the integral to find the volume of the solid generated by revolving R about the following axis using indicated methods. No credits using different methods.

(a) the y -axis (using cylindrical shells).

(b) the x -axis (using disks).

Bonus

12 (4 points) Show that the equation $10x + \cos x = 0$ has exactly one real root.

13 (4 points) The base of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is decreasing at a rate of $10 \text{ cm}^2/\text{min}$. At what rate is the height changing when the base is 20 cm and height is 10 cm?