MATH 16500 Final Exam

Exam is 7 pages plus cover page. Follow the instructions for each question. Show enough of your work that we can understand what you are doing.

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1 (12 points) Determine the infinite limit. $\lim_{x \to 2^+} \dfrac{x^2 - 2x - 3}{x - 2}$.

2 (12 pints) Evaluate the limit, if it exists. $\lim_{x \to 2} \dfrac{\frac{1}{x^2} - \frac{1}{4}}{x - 2}$.

3 (12 points) Given $f(x) = \sqrt{x+1}$, find $f'(a)$ using $f'(a) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$. No credits if you use any other formulas.

4 (12 points) Differentiate $f(\theta) = \dfrac{\cos(2\theta)}{\sin\theta + \cos\theta}$ with respect to θ.

5 (12 points) Find the derivative of $y = \sqrt{x}(1+x)^2$ with respect to x.

6 (12 points) Given $xy^2 = y\sin(x) - 1$. Find dy/dx by implicit differentiation.

7 (12 points) Find the linearization $L(x)$ of the function $f(x) = \dfrac{x-1}{x+1}$ at $a = 4$.

8 (14 points) Let $f(x) = x^{4/3} - 4x^{1/3}$. Find the critical numbers of f.

9 (14 points) Given $y = \dfrac{x+1}{x-2}$. Sketch the graph indicating the local maximum/minimum points, and points of inflection.

10 (14 points) If $1200\pi\ cm^2$ of the material is available to make a circular cylindrical tank with an open top, find the largest possible volume of the tank.

11 (12 points) Express the integral as a limit of Riemann sums, $\int_0^1 x^3\sqrt{1+x^2}\,dx$.

12 (12 points) Evaluate the integral $\int_0^1 6u(1+u^2)^2\,du$.

13 (12 points) Evaluate the indefinite integral $\int \frac{(1+\sqrt{x})^2}{\sqrt{x}}\,dx$.

14 (14 points) Sketch the region enclosed by the given curves and find the area. $y = 4 - x^2,\ y = x - 2$.

15 (12 points) Set up an integral for the volume of the solid obtained by rotating the region bounded by the curves $x = y^2, x = 1$ about the y-axis.

16 (12 points) A variable force of $F = x(4 - x)$ pounds moves an object along a straight line when it is x feet from the origin. Calculate the work done in moving the object from $x = 0$ ft to $x = 4$ ft.

17 (Bonus, 8 points) Prove the statement using the ϵ, δ definition of a limit. $\lim_{x \to 2} \frac{3x - 2}{4} = 1$.

18 (Bonus, 8 points) Use the method of cylindrical shells to set up an integral for the volume generated by rotating the region bounded by $y = x^{1/3}; y = 0; x = 1$ about the $y = 1$.