

MATH 16500 Final Exam

Exam is 10 pages plus cover page. Follow the instructions for each question. Show enough of your work that we can understand what you are doing.

1 (4 points) Use the definition of derivative to show that the derivative of $f(x) = \frac{1}{x}$ is $f'(x) = -\frac{1}{x^2}$.

2 (6 points) Let $f(x) = x^{2/3}(2x - 5)$. Find the absolute maximum value and the absolute minimum value of f on $[0, 8]$.

3 (12 points) Find the limit if it exists. Explain why if it does not exist.

(a) $\lim_{x \rightarrow 2^-} \frac{x+2}{x-2}$.

(b) $\lim_{x \rightarrow -1} \frac{x^2 + 4x + 3}{x^2 - x - 2}$

(c) $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1}$

(d) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(1 + 2\frac{i}{n}\right) \frac{1}{n}$.

4 (12 points) Compute the derivatives of the following functions.

(a) $f(x) = \frac{(1 + 2x)^2}{\sqrt{x}}$

(b) $g(x) = (x^2 + 3)^3$

(c) $f(x) = \int_1^x t\sqrt{1+t^2} dt$

5 (6 points) Use implicit differentiation to find an equation of the tangent line to the curve $2x^2y = y^3 + x^3$ at $(1, 1)$.

6 (6 points) Sketch the region enclosed by the curves: $y = x^2 - 1$ and $y = 1 - x$. Find the area of the region.

7 (12 points) Let $f(x) = \frac{x+2}{x-1}$.

- (a) find critical numbers,
- (b) determine intervals on which f is increasing or decreasing,
- (c) find local maximum values and local minimum values,
- (d) determine the intervals on which the graph is concave up or concave down,
- (e) Sketch the graph.

8 (8 points) A rectangular storage container with an open top is to have a volume 8 cubic meters. The length of its base is twice the width. Material for the base costs \$3 per square meter. Material for sides cost \$4 per square meter. Find the cost of materials for the cheapest such container.

9 (6 points) Express the integral as a limit of Riemann sums. $\int_0^2 x\sqrt{x^2+1} dx$. Do not evaluate the limit.

10 (16 points) Evaluate the following integrals:

(a) $\int 2(x+1)(x^2+2x)dx.$

(b) $\int_0^1 \frac{x}{\sqrt{1+x^2}}dx.$

(c) $\int \frac{(x+2)^2}{x^4}dx.$

(d) $\int x \cos(x^2)dx.$

11 (8 points) Let A be the region bounded by the graphs of $y = x(4 - x^2)$, $0 \leq x \leq 2$ and $y = 0$. Set up (but do not compute) the integral to find the volume of the solid generated by revolving A about

(a) the y -axis.

(b) the line $y = -1$.

Indicate the methods you use.

12 (6 points) Find the average value of $f(x) = x^2(4 - x)$ on $[0, 4]$.

Bonus

13 (4 points) The altitude of triangle is increasing at a rate of 3 cm/min while the area of the triangle is increasing at a rate of 5 cm²/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 25 cm²?

14 (4 points) Evaluate $\int \frac{x}{(x+2)^3} dx$.