Holomorphic foliations and complex polynomial vector fields

ABSTRACT:

Given a vector field in an open set $U \subset \mathbb{R}^n$, it defines a system of ODEs whose solutions are the trajectories of the vector field. To globally understand their behavior, it is necessary to study the singular points (zeroes of the vector field), and so it becomes useful to consider the problem in the complex domain.

In general, a holomorphic foliation of dimension 1 in a complex manifold $M$ is a decomposition of $M$ as a disjoint union of Riemann surfaces. Such foliations are locally defined by holomorphic vector fields. In many cases, for example, if $M$ is a projective space, we need to allow for singularities. In the study of foliations, a variety of techniques (analytic, topological, geometric...) prove to be useful.

In this talk, we focus on dimension 1 foliations in complex surfaces, and show how they allow one to study problems related to vector fields in the complex affine plane. We will conclude by briefly presenting some results on vector fields with simply connected trajectories that are obtained in collaboration with A. Bustinduy (Nebrija Univ. Madrid).