Department of Mathematical Sciences welcomes

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Mastodon Theorem – 20 Years in the Making

November 8, 2019

Hosted by:
Prof. Maxim Yattselev

Tea begins at 3:00
in LD 259

Research Topic
begins at 3:30
in LD 229

ABSTRACT:

The *Mastodon theorem* (PD., D. Legg, D. Townsend, 2002), establishes that the regular bi-pyramid (North and South poles, and an equilateral triangle on the Equator) is the unique up to rotation five-point configuration on the sphere that maximizes the product of all mutual distances. More generally, given a configuration of points $\{x_1, \ldots, x_N\}$ on the unit sphere in $\mathbb{S}^{n-1} \subseteq \mathbb{R}^n$, its *Riesz s-energy* is defined as

$$\sum_{1\le i<j\le N} \frac{1}{\|x_i - x_j\|^s}, \quad s>0; \qquad \sum_{1\le i<j\le N} \log \frac{1}{\|x_i - x_j\|}, \quad s=0.$$

The regular bi-pyramid minimizes the *logarithmic energy* ($s = 0$ case) for five points on $\mathbb{S}^2$. Optimal point configurations that minimize the s-energy have broad applications in sciences, economics, information theory, etc. Rigorous proofs of optimality are extremely hard though. Even the important Coulomb energy ($s = 1$) case for five points on the unit sphere in 3-D space was resolved only recently (2013) by Richard Schwartz utilizing a computer-aided proof. In a subsequent monograph Schwartz extends the optimality of the bipyramid to all $s < s^*$. In a joint work with Oleg Musin we generalize the Mastodon Theorem to $n + 2$ points on $\mathbb{S}^{n-1}$, namely we characterize all stationary configurations, and show that all local minima occur when a configuration splits in two orthogonal simplexes of k and ℓ vertices, $k + \ell = n + 2$, with global minimum attained when $k = \ell$ or $k = \ell + 1$ depending on the parity of n.

ABOUT THE SPEAKER:

Peter Dragnev studied at Sofia State University and the Institute of Mathematics of the Bulgarian Academy of Sciences. He received a Ph.D. from the University of South Florida in 1997. He is a full professor at Purdue University Fort Wayne and currently serves as head of the department of mathematical sciences. His research interests are in analysis, in particular, approximation theory and potential theory.

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