Equilibrium of discrete charges on the real line

ABSTRACT:

Stieltjes (1885) found the first description of the equilibrium position for a system of discrete charges on a segment of the real line, characterizing their locations by the zeros of Jacobi polynomials. Schur, Fekete, Siegel and others developed these ideas in connection with various applications in analysis and number theory. We consider two related types of the minimum energy (equilibrium) problems for charges on the real line. In the first case, the equilibrium is attained in the external field of a charge located outside the real line. In the second case, we study the equilibrium of charges with a fixed value of potential at a given point off the real line. The equilibrium position of charges is described by the zeros of two explicit families of polynomials: One family has a particularly simple expression as a linear combination of powers of two linear functions, while the other family is related to generalized Jacobi polynomials. We also find explicit formulas for the minimum energy. Our results have applications to some questions of Erdős about the largest disks contained in polynomial lemniscates. This is joint work with Artūras Dubickas (Vilnius University).

ABOUT THE SPEAKER:

Igor Pritsker received his PhD from the University of South Florida, Tampa, in 1995. He joined the Department of Mathematics at Oklahoma State University in 1999, and was promoted to the rank of Professor in 2007. Igor’s research interests include Analytic Number Theory, Approximation Theory, Complex Analysis, Potential Theory and Probability Theory. He has received the Ralph E. Powe Award, Humboldt Research Fellowship, several NSF and NSA grants, and served as AT & T professor at OSU. Igor Pritsker was recently appointed the Vaughn Foundation Professor in Number Theory.