

MATH M118: Finite Mathematics
Practice Department Final Examination: Part B (Chapters 5, 6, 7, and 9)

1. There are 12 students in a small section of Math 118. Their scores from Test 4 are listed below. Find the mean, median, and mode.

78, 82, 80, 76, 100, 91, 52, 66, 80, 75, 94, 61

2. Given the following values of a random variable X , find $E[X]$ and the standard deviation of X .

X	$\text{Pr}[X]$
1	0.15
2	0.20
3	0.30
5	0.35

3. A pocket contains 3 nickels, 2 dimes, and 1 quarter. Two coins are selected simultaneously and at random. A random variable, X , is defined to be the total value in cents of the two coins. Find the expected value, $E[X]$.

4. A piggy bank containing 500 unfair coins is dumped onto a table. The probability of tails is 0.8 on every coin. Let the random variable X be the number of tails that appear.

(a) Find the expected value of X .

(b) Find the standard deviation of X . Express your answer as a decimal number with two decimal places.

5. Let X be a normal random variable with mean 410 and standard deviation 20. Find $\text{Pr}[360 \leq X \leq 420]$. Give four decimal places in your answer.

6. Assume that the length of angelfish is a normal random variable with mean length of 10.2 cm and standard deviation of 2.1 cm. Find the probability that a randomly selected angelfish will be at least 7 cm in length.

7. Assume that IQ scores are normally distributed with a mean of 100 and a standard deviation of 15.

(a) If people are identified for special education when their IQ score is in the lowest 9 percent of the population, what IQ score (to the nearest whole number) would a person need to receive special education?

(b) What IQ score (to the nearest whole number) does 44% of the population exceed?

8. A champion archer has a long-established probability of hitting the bulls-eye 80 percent of the time. If, in a particular shooting session, 200 attempts are made, what is the probability that he hits the bulls-eye. . .

(Hint: use the normal approximation to the binomial random variable)

- (a) between 165 and 180 times, inclusively?
- (b) less than or equal to 170 times?

9. Let A be a 5×7 matrix, let B be a 7×7 matrix, and let C be a 7×5 matrix. Which of the following is NOT defined?

- (a) BC
- (b) AC
- (c) AB
- (d) CA
- (e) BA
- (f) They're all defined

10. Given the matrices below, find the products AB , BC , AC , and CD , if they are defined.

$$A = \begin{bmatrix} 3 & -7 \\ 7 & -4 \end{bmatrix} \quad B = \begin{bmatrix} -1 & -2 & 1 \\ -6 & -5 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 4 \\ 0 & -3 \\ -1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

11. Given matrices E and F , find $2E - F$.

$$E = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix} \quad F = \begin{bmatrix} 5 & 0 \\ -1 & 2 \end{bmatrix}$$

12. Solve the following system of equations.

$$\begin{aligned} x - 2y + 3z &= 4 \\ 4x + y + z &= 1 \\ 2x - y + z &= -1 \end{aligned}$$

13. One day a business sold 45 pairs of gloves. The cloth gloves sold for \$8.95 per pair and the leather gloves sold for \$16.25 per pair. The business took in \$504.95. How many of each kind were sold?

14. Graph $2x + 3y \leq 6$

15.

Graph the solution set of the following system of inequalities, or constraints, and find corner points, if they exist.

$$x + y \geq 1$$

$$-x + y \leq 2$$

$$x \leq 4$$

#16-17: Find the indicated optimal values and the points at which they occur. Show the graphs and work for each problem.

16.

Maximize $f = 2x + y$ subject to

$$3x + 5y \leq 15,$$

$$3x + 2y \leq 12,$$

$$x, y \geq 0$$

17.

Minimize $T = x + 3y + 10$ subject to

$$2x + 4y \geq 8,$$

$$4x + 3y \geq 12,$$

$$x, y \geq 0$$

18. A manufacturing company makes two types of water skis, X and Y. Type X requires 6 labor-hours for fabricating and 1 labor-hour for finishing. Type Y requires 4 labor-hours for fabricating and 1 labor-hour for finishing. The maximum labor-hours for fabricating is 108 hours and for finishing is 24 hours.

If the profit on Type X is \$40 and the profit on Type Y is \$30, how many of each type of ski should be manufactured to achieve a maximum profit? What is the maximum profit?

19. A Markov chain has the following transition matrix:

$$\begin{array}{c} \begin{array}{cc} & \begin{array}{cc} X & Y \end{array} \\ \begin{array}{c} X \\ Y \end{array} & \begin{bmatrix} .8 & .2 \\ .3 & .7 \end{bmatrix} \end{array}$$

If the chain begins in state Y, what is the probability that it will be in state X after three transitions? Show answer to three decimal places.

20. Kylie shops exclusively at Circle Center and Keystone. If she shops at Keystone, the probability that she will shop next time at Circle Center is 0.6. If she shops at Circle Center, the probability that she will shop at Keystone next time is 0.5. If she last shopped at Keystone, what is the probability that she will shop at Keystone again two trips from now?

21. For the given transition matrix and state vector, find the state vector:

$$T = \begin{bmatrix} .6 & .4 \\ .2 & .8 \end{bmatrix} \quad P_0 = [.7 \quad .3]$$

- (a) one transition later
 - (b) three transitions later
22. Of voters sampled, 60% of the Republicans (that is, those who voted Republican in the last election) will vote Republican in the next election, 20% will vote Democrat, and 20% will vote Independent. Of the Democrats, 10% will vote Republican and 80% will vote Democrat and 10% will vote Independent. Of the Independents, 40% will vote Republican, 20% will vote Democrat, and 40% will vote Independent.
- (a) What is the transition matrix for this Markov Chain?
 - (b) If in the most current election, 55% voted Republican, 42% voted Democrat, and 3% voted Independent, what will the distribution of voters be two elections from now?