Problem 1. Let $A, B$ and $C$ be measurable sets on the line such that
\[ m(A \Delta B) + m(B \Delta C) = m(A \Delta C), \]
where $m$ is the Lebesgue measure and as usual,
\[ A \Delta B = (A \setminus B) \cup (B \setminus A). \]
Prove that
\[ m[B \setminus (A \cup C)] = 0. \]

Problem 2. Let $E$ be a measurable set on the line and $f$ a function on $E$ such that the set
\[ \{ x \in E \mid f(x) < a \sin x \} \]
is measurable for any real $a$. Prove that the function $f$ is measurable.

Problem 3. Let us enumerate all rational numbers on $[0, 1]$, $\mathbb{Q} \cap [0, 1] = \{ r_1, r_2, \ldots \}$. Prove that the function
\[ f(x) = \sum_{k=1}^{\infty} \frac{|x - r_k|^{1/2}}{k^{3/2}} \]
is differentiable almost everywhere on $[0, 1]$.

Problem 4. Let $f \in L^2[0, 1]$. Prove that
\[ \left( \int_0^1 |f(x) - \int_0^1 f(x) \, dx| \, dx \right)^2 \leq \int_0^1 f^2(x) \, dx - \left( \int_0^1 f(x) \, dx \right)^2. \]

Problem 5. Let $T(f)$ be a bounded linear functional on $L^2[0, 1]$ such that $T(x^n) = \frac{1}{n+1}$ for $n = 0, 1, 2, \ldots$. Prove that
\[ T(f) = \int_{[0,1]} f(x) \, dx \]
for all $f \in L^2[0, 1]$.

Problem 6. Prove that the sequence of functions,
\[ f_n(x) = \sum_{k=1}^{n} \frac{\sin(kx)}{k^{3/6}}, \quad n = 1, 2, \ldots, \]
is a Cauchy sequence in $L^2[0, 2\pi]$. 