1. a) Find the general solution of the equation \( xz_x - yz_y = x^2/z \) in the domain \( z < 0 \).
   b) Find the solution of the equation that satisfy the initial condition \( z(x, x) = -1 \).

2. Consider the initial value problem for \( t \geq 0 \):
   \[ z_t - z^2z_x = 0, \quad z(x, 0) = e^{-x^2}. \]
   a) Find \( t_c \), the value of \( t \) when the shock develops.
   b) Sketch the graph of the solution \( z(x, t) \) at \( t = 0 \) and at \( t = t_c \). The graphs should be sufficiently accurate. At the graph of \( z(x, t_c) \), point out the feature that indicate the developing of the shock.
   c) Find the maximum value of \( z(x, t) \) as \(-\infty < x < \infty, \ 0 \leq t < t_c \). (An analytic formula for the solution \( z(x, t) \) cannot be found in elementary functions.)

3. Consider the boundary value problem in the unit disk:
   \[ \Delta u(x, y) = 1, \quad x^2 + y^2 < 1, \]
   \[ \frac{\partial u}{\partial n}(x, y) = \lambda, \quad x^2 + y^2 = 1, \]
   where \( \lambda \) is a real number.
   a) Show that if \( u_1(x, y) \) and \( u_2(x, y) \) are two solutions in \( C^2(\Omega) \) of the boundary value problem, then \( u_1 - u_2 \) is a constant function.
   b) Find all values of \( \lambda \) such that the boundary value problem has a solution in \( C^2(\Omega) \).
   c) For the values of \( \lambda \) such that the boundary value problem has a solution in \( C^2(\Omega) \), solve the boundary value problem, and prove that you have found all solutions of the boundary value problem.

4. Use separation of variables to solve the initial value problem on the line \(-\infty < x < \infty \):
   \[ \left\{ \begin{array}{l}
   u_{tt}(x, t) = u_{xx}(x, t) + u(x, t), \\
   u(x, 0) = 2\sin^2x, \\
   u_t(x, 0) = \sin x.
   \end{array} \right. \]

5. Let \( \Omega \) be a bounded normal domain in \( \mathbb{R}^3 \). Suppose \( u \in C^2(\Omega \times \mathbb{R}_{\geq 0}) \) is such that
   \[ u_t(x, t) = \Delta u(x, t) - \sin(u(x, t)), \quad x \in \Omega, \]
   \[ u(x, t) = 0, \quad x \in \partial \Omega. \quad (1) \]
   a) Let \( E(t) = \int_{\Omega} \left( |\grad u(x, t)|^2 - \cos(u_t(x, t)) \right) d^3x \). Show that \( \frac{dE}{dt} \leq 0 \).
   b) Use part a) to show that there is a unique function \( u \in C^2(\Omega \times \mathbb{R}_{\geq 0}) \) satisfying (1) and such that \( u(x, 0) = 0 \).