

**Math M119 Brief Survey of Calculus  
Practice DEPARTMENTAL FINAL EXAMINATION**

- **DO NOT OPEN** this test booklet until you are instructed to do so.
- There are 35 questions.
- **PRINT** your name and student ID# and check your section below.
- You have two hours to complete this examination.
- No scratch paper – if you need extra paper use the back of the cover sheet.
- Sharing calculators is not permitted.
- The only permissible calculator is the TI-**30XA**

**NO notes, books; Cell phones should be OFF. Earpieces are not permitted.**

**To receive credit, show supporting work.**

**Unless otherwise indicated:**

- **give exact answers (not approximations)**
- **simplify your answers**

<b>NAME</b> (Print Clearly)	
<b>UNIV ID#</b>	

1. Given  $y = \frac{3}{x^4} + 2x^4$  find  $\frac{dy}{dx}$

2. Find the instantaneous rate of change of  $f(x) = \ln(x^2 + 3x)$  at  $x = 1$ .

3. Differentiate:  $y = e^{-x} \cdot x^5$

4. Find the slope of the line tangent to the graph of  $y = 8\sqrt{x}$  at  $x = 25$ .

5. If  $f(x) = 3x^2(4x + 1)^5$  find  $f'(x)$ . Simplify your answer by multiplying constants together.

6. If  $g(x) = 2e^{3x}$  find  $g''(0)$ .

7. If  $y = 3 \ln 2x$ , find  $\frac{d^2y}{dx^2}$ .

8. Write an equation of the line tangent to the curve  $y = f(x) = x^3 + 1$  at  $x = -2$ .

9. How long will it take an account to double in value if interest is compounded continuously at 3.2% per year? (1 decimal place)

10. If a radioactive substance has a half-life of 42 years, what is the continuous annual decay rate? Give your answer as a percent with 2 decimal places.

11. A town's population is currently 1,600 and is growing at a continuous annual rate of 1.9% per year.

a. How long will it take for the population to reach 2,000? (1 decimal place)

b. Find the rate of change of the population after 7 years. (round to the nearest person per year)

12. If we know that  $\left. \frac{dy}{dx} \right|_{x=3} = -2$ , then which one of the following is true?
- A. The function is increasing at  $x=3$
  - B. The function is decreasing at  $x=3$
  - C. The function is concave up at  $x=3$
  - D. The function is concave down at  $x=3$
  - E. None of the above
13. If we know that  $f''(1) = 4$ , then which one of the following is true?
- A. The function is increasing at  $x=1$
  - B. The function is decreasing at  $x=1$
  - C. The function is concave up at  $x=1$
  - D. The function is concave down at  $x=1$
  - E. None of the above
14. For the function  $f(x) = \frac{1}{3}x^3 + x^2 - 8x$ , the factored derivative is  $f'(x) = (x + 4)(x - 2)$ . Which of the following is true?
- A. There is a relative maximum at  $x = -2$  and a relative minimum at  $x = 4$
  - B. There is a relative maximum at  $x = 2$  and a relative minimum at  $x = -4$
  - C. There is a relative maximum at  $x = 4$  and a relative minimum at  $x = -2$
  - D. There is a relative maximum at  $x = -4$  and a relative minimum at  $x = 2$
  - E. None of the above.
15. If we know that  $f'(2) = 0$  and  $f''(2) = -5$ , then which of the following is true?
- A. There is a relative minimum at  $x = 0$
  - B. There is a relative maximum at  $x = -5$
  - C. There is a relative minimum at  $x = 2$
  - D. There is a relative maximum at  $x = 2$
  - E. None of the above.

16. Consider the function  $f(x) = x^4 - 4x^3$ . How many inflection points are there?
17. Suppose that  $C$  is a total cost function. If  $C(50) = \$150$  and if  $C'(50) = \$2.40$ , estimate  $C(53)$ . This is the total cost to produce 53 items.
18. The profit in dollars from the sale of  $x$  chairs is  $P(x) = x^3 - 9x^2 + 8x + 5$ . Find the marginal profit when  $x = 10$ .
19. Consider the polynomial  $y = f(x) = x^3 - 6x^2$  on the closed interval  $[-1, 7]$ . Find the absolute maximum and the absolute minimum values for the function on the interval  $[-1, 7]$ .

20. A state park charges \$100 for an annual pass. At this rate 750 people purchase passes every year. For each \$5 decrease in price 15 more people purchase a pass. What price should the park charge in order to maximize **revenue**? Use calculus and show your work.

21. The demand function for suits is given by  $p = 156 - 0.4x$  and we also know that the total cost to produce  $x$  suits is  $C(x) = 4000 + 0.25x^2$ . How many suits should be sold in order to maximize profit?

Find each indefinite integral:

22.  $\int \left( \frac{4}{x^3} + \frac{7}{x} \right) dx$

23.  $\int (2e^{5x} + 1) dx$

24.  $\int \left( \frac{1}{\sqrt{x}} - 5x^6 \right) dx$

25. Find  $f(x)$  such that  $f'(x) = 9x^2 + 4x - 5$  and  $f(0) = 3$ .



26. Rock Industries finds that the marginal cost of producing the  $x^{\text{th}}$  climbing harness is  $C'(x) = x^3 - x$ . Find the total cost to produce 40 harnesses, assuming that fixed costs are \$6500.

27. Approximate the area under the graph of  $f(x) = x^2 + 1$  over the interval  $[0, 4]$ . Use the left-hand sum and compute the area of 4 rectangles.

28. Now find the exact area between the graph of  $f(x) = x^2 + 1$  and the x-axis on the interval  $[0, 4]$ .

29. Evaluate the definite integral:  $\int_1^2 (4t^3 - 1) dt$

30. Find the area between the graph of  $f(x) = \sqrt{x}$  and the x-axis on the interval  $[4, 25]$ .

31. Larry's Lawncare estimates that its sales are growing continuously at a rate given by  $S'(t) = 10e^t$  where  $S'(t)$  is given in dollars per day, on day  $t$ . Find the accumulated sales for the first 5 days. (nearest cent)

Evaluate each definite integral. Then choose the best description of the area of the region involved.

- a. More area is above the x-axis than is below.
- b. More area is below the x-axis than is above.
- c. The areas above and below the x-axis are equal.

32.  $\int_{-1}^1 (x^3 + 4x) dx$

33.  $\int_0^2 (x^2 - x) dx$

34. Find the present value of \$10,000 due 8 years in the future, if interest is compounded continuously at an annual rate of 4%. (nearest cent)

35. Find the accumulated present value of a continuous income stream of \$10,000 per year for 8 years, if interest is compounded continuously at an annual rate of 4%. (nearest dollar)

M119 Practice Final Exam ANSWERS

1.  $-12x^{-5} + 8x^3$  or  $-\frac{12}{x^5} + 8x^3$
2.  $5/4$
3.  $5x^4e^{-x} - x^5e^{-x}$
4.  $4/5$
5.  $60x^2(4x + 1)^4 + 6x(4x + 1)^5$
6. 18
7.  $-\frac{3}{x^2}$
8.  $y = 12x + 17$
9. 21.7 years
10. 1.65%
- 11a. 11.7 years
- 11b. approx. 35 people per year
12. B
13. C
14. D
15. D
16. 2
17. \$157.20
18. \$128
19. absolute max value is 49; absolute min value is -32
20. \$175
21. 120
22.  $-2x^{-2} + 7 \ln x + C$  or  $-\frac{2}{x^2} + 7 \ln x + C$
23.  $\frac{2}{5}e^{5x} + x + C$
24.  $2\sqrt{x} - \frac{5}{7}x^7 + C$
25.  $3x^3 + 2x^2 - 5x + 3$
26. \$645,700
27. 18
28.  $25\frac{1}{3}$  or  $\frac{76}{3}$
29. 14
30. 78
31. \$1474.13
32. 0 c
33.  $2/3$  a
34. \$7261.49
35. \$68,463