DO NOT OPEN this test booklet until you are instructed to do so.
There are 35 questions.
PRINT your name and student ID# and check your section below.
You have two hours to complete this examination.
No scratch paper – if you need extra paper use the back of the cover sheet.
Sharing calculators is not permitted.
The only permissible calculator is the TI-30XA

NO notes, books; Cell phones should be OFF. Earpieces are not permitted.

To receive credit, show supporting work.
Unless otherwise indicated:
• give exact answers (not approximations)
• simplify your answers

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1. Given \( y = \frac{3}{x^4} + 2x^4 \) find \( \frac{dy}{dx} \)

2. Find the instantaneous rate of change of \( f(x) = \ln(x^2 + 3x) \) at \( x = 1 \).

3. Differentiate: \( y = e^{-x} \cdot x^5 \)

4. Find the slope of the line tangent to the graph of \( y = 8\sqrt{x} \) at \( x = 25 \).
5. If \( f(x) = 3x^2(4x + 1)^5 \) find \( f'(x) \). Simplify your answer by multiplying constants together.

6. If \( g(x) = 2e^{3x} \) find \( g''(0) \).

7. If \( y = 3 \ln 2x \), find \( \frac{d^2y}{dx^2} \).

8. Write an equation of the line tangent to the curve \( y = f(x) = x^3 + 1 \) at \( x = -2 \).
9. How long will it take an account to double in value if interest is compounded continuously at 3.2% per year? (1 decimal place)

10. If a radioactive substance has a half-life of 42 years, what is the continuous annual decay rate? Give your answer as a percent with 2 decimal places.

11. A town’s population is currently 1,600 and is growing at a continuous annual rate of 1.9% per year.
   a. How long will it take for the population to reach 2,000? (1 decimal place)
   b. Find the rate of change of the population after 7 years. (round to the nearest person per year)
12. If we know that \( \frac{dy}{dx} \bigg|_{x=3} = -2 \), then which one of the following is true?

A. The function is increasing at x=3
B. The function is decreasing at x=3
C. The function is concave up at x=3
D. The function is concave down at x=3
E. None of the above

13. If we know that \( f''(1) = 4 \), then which one of the following is true?

A. The function is increasing at x=1
B. The function is decreasing at x=1
C. The function is concave up at x=1
D. The function is concave down at x=1
E. None of the above

14. For the function \( f(x) = \frac{1}{3} x^3 + x^2 - 8x \), the factored derivative is \( f'(x) = (x + 4)(x - 2) \). Which of the following is true?

A. There is a relative maximum at \( x = -2 \) and a relative minimum at \( x = 4 \)
B. There is a relative maximum at \( x = 2 \) and a relative minimum at \( x = -4 \)
C. There is a relative maximum at \( x = 4 \) and a relative minimum at \( x = -2 \)
D. There is a relative maximum at \( x = -4 \) and a relative minimum at \( x = 2 \)
E. None of the above

15. If we know that \( f'(2) = 0 \) and \( f''(2) = -5 \), then which of the following is true?

A. There is a relative minimum at \( x = 0 \)
B. There is a relative maximum at \( x = -5 \)
C. There is a relative minimum at \( x = 2 \)
D. There is a relative maximum at \( x = 2 \)
E. None of the above.
16. Consider the function \( f(x) = x^4 - 4x^3 \). How many inflection points are there?

17. Suppose that \( C \) is a total cost function. If \( C(50) = $150 \) and if \( C'(50) = $2.40 \), estimate \( C(53) \). This is the total cost to produce 53 items.

18. The profit in dollars from the sale of \( x \) chairs is \( P(x) = x^3 - 9x^2 + 8x + 5 \). Find the marginal profit when \( x = 10 \).

19. Consider the polynomial \( y = f(x) = x^3 - 6x^2 \) on the closed interval \([-1, 7]\). Find the absolute maximum and the absolute minimum values for the function on the interval \([-1, 7]\).
20. A state park charges $100 for an annual pass. At this rate 750 people purchase passes every year. For each $5 decrease in price 15 more people purchase a pass. What price should the park charge in order to maximize revenue? Use calculus and show your work.

21. The demand function for suits is given by \( p = 156 - 0.4x \) and we also know that the total cost to produce \( x \) suits is \( C(x) = 4000 + 0.25x^2 \). How many suits should be sold in order to maximize profit?
Find each indefinite integral:

22. \( \int \left( \frac{4}{x^3} + \frac{7}{x} \right) dx \)

23. \( \int (2e^{5x} + 1) dx \)

24. \( \int \left( \frac{1}{\sqrt{x}} - 5x^6 \right) dx \)

25. Find \( f(x) \) such that \( f'(x) = 9x^2 + 4x - 5 \) and \( f(0) = 3 \).
26. Rock Industries finds that the marginal cost of producing the \( x \)th climbing harness is \( C'(x) = x^3 - x \). Find the total cost to produce 40 harnesses, assuming that fixed costs are $6500.

27. Approximate the area under the graph of \( f(x) = x^2 + 1 \) over the interval \([0, 4]\). Use the left-hand sum and compute the area of 4 rectangles.

28. Now find the exact area between the graph of \( f(x) = x^2 + 1 \) and the x-axis on the interval \([0, 4]\).

29. Evaluate the definite integral: \( \int_1^2 (4t^3 - 1) \, dt \)
30. Find the area between the graph of \( f(x) = \sqrt{x} \) and the x-axis on the interval \([4, 25]\).

31. Larry’s Lawncare estimates that its sales are growing continuously at a rate given by \( S'(t) = 10e^t \) where \( S'(t) \) is given in dollars per day, on day \( t \). Find the accumulated sales for the first 5 days. (nearest cent)

Evaluate each definite integral. Then choose the best description of the area of the region involved.

a. More area is above the x-axis than is below.
b. More area is below the x-axis than is above.
c. The areas above and below the x-axis are equal.

32. \( \int_{-1}^{1} (x^3 + 4x) \, dx \)  
33. \( \int_{0}^{2} (x^2 - x) \, dx \)
34. Find the present value of $10,000 due 8 years in the future, if interest is compounded continuously at an annual rate of 4%. (nearest cent)

35. Find the accumulated present value of a continuous income stream of $10,000 per year for 8 years, if interest is compounded continuously at an annual rate of 4%. (nearest dollar)
M119 Practice Final Exam ANSWERS

1. \(-12x^{-5} + 8x^3\) or \(-\frac{12}{x^5} + 8x^3\)
2. 5/4
3. \(5x^4e^{-x} - x^5e^{-x}\)
4. 4/5
5. \(60x^2(4x + 1)^4 + 6x(4x + 1)^5\)
6. 18
7. \(-\frac{3}{x^2}\)
8. \(y = 12x + 17\)
9. 21.7 years
10. 1.65%
11a. 11.7 years
11b. approx. 35 people per year
12. B
13. C
14. D
15. D
16. 2
17. $157.20
18. $128
19. absolute max value is 49; absolute min value is -32
20. $175
21. 120
22. \(-2x^{-2} + 7 \ln x + C\) or \(-\frac{2}{x^2} + 7 \ln x + C\)
23. \(\frac{2}{5}e^{5x} + x + C\)
24. \(2\sqrt{x} - \frac{5}{7}x^7 + C\)
25. \(3x^3 + 2x^2 - 5x + 3\)
26. $645,700
27. 18
28. \(25\frac{1}{3}\) or \(\frac{76}{3}\)
29. 14
30. 78
31. $1474.13
32. 0 c
33. 2/3 a
34. $7261.49
35. $68,463