1. (10 points) Prove that the function \( f(z) = |z^2| \) has a complex derivative at \( z = 0 \), and at no other point.

2. (10 points) Compute the improper integral below, justifying your methodology:
\[
\int_{-\infty}^{\infty} \frac{x^2}{x^4 + 5x^2 + 4} \, dx
\]

3. (10 points) Show that if \( |b| < 1 \), then the polynomial \( z^3 + 3z^2 + bz + b^2 \) has exactly two roots (counted with multiplicity) in the unit disk.

4. (10 points) Find a conformal map that takes the region \( \{|z| < 1, |z - 1/2| > 1/2\} \) in a one-to-one fashion onto the unit disk.

5. (10 points) Prove that if \( f \) is entire and satisfies \( |f(z^2)| \leq |f(z)| \) for all \( z \in \mathbb{C} \), then \( f \) is constant.

6. (10 points) Show that the function \( \frac{1}{2}(z + 1/z) \) maps the half-disk \( \{|z| < 1, \text{Im} \, z < 0\} \) in a one-to-one fashion onto the upper half-plane \( \{\text{Im} \, z > 0\} \).

7. (10 points) Let \( \mathbb{D} \) denote the unit disk, and suppose the holomorphic function \( g : \mathbb{D} \rightarrow \mathbb{D} \) has two distinct fixed points. Prove that \( g(z) \equiv z \).